TD Baseband Filtering

Exercice 1 - RC filter

We have the following RC filter :

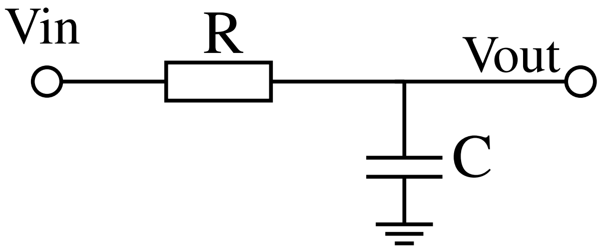

FIGURE 1 – RC passive filter

Question 1.1 Calculate the transfer function $T(p) = \frac{V_{out}(p)}{V_{in}(p)}$

Réponse 1.1 $T(p) = \frac{1}{1+R\,C\,p}$

Question 1.2 Is the filter stable ?

Réponse 1.2 To determine if the filter is stable, we need to calculate the poles of $T(p)$ and verify that all of them have a negative real part. In this case, we have only one pole equal to $-\frac{1}{R\,C}$. So the filter is stable

Question 1.3 Determine the attenuation and the group delay expression.

Réponse 1.3 The attenuation in dB is given by $Att_{dB}(\omega) = -20\log_{10}(\mid T(\omega)\mid)$

$$T(j\omega) = \frac{1}{1 + R\,C\,j\,\omega}$$

$$\mid T(\omega)\mid = \frac{1}{\sqrt{(1 + R^2\,C^2\,\omega^2)}}$$

$$Att_{dB}(\omega) = -20\log_{10}\left(\frac{1}{\sqrt{(1 + R^2\,C^2\,\omega^2)}}\right) = 10\log_{10}(1 + R^2\,C^2\,\omega^2)$$

The group delay is given by : $t_g(\omega) = -\frac{\partial \arg[T(j\,\omega)]}{\partial\omega}$

$$\arg[T(j\,\omega) = -arctan(R\,C\,\omega)$$

$$t_g = -\frac{\partial \arg[T(j\,\omega)]}{\partial\omega} = \frac{R\,C}{1 + R^2C^2\omega^2}$$

Question 1.4 Trace them for an R = 10 kΩ and C= 1 nF

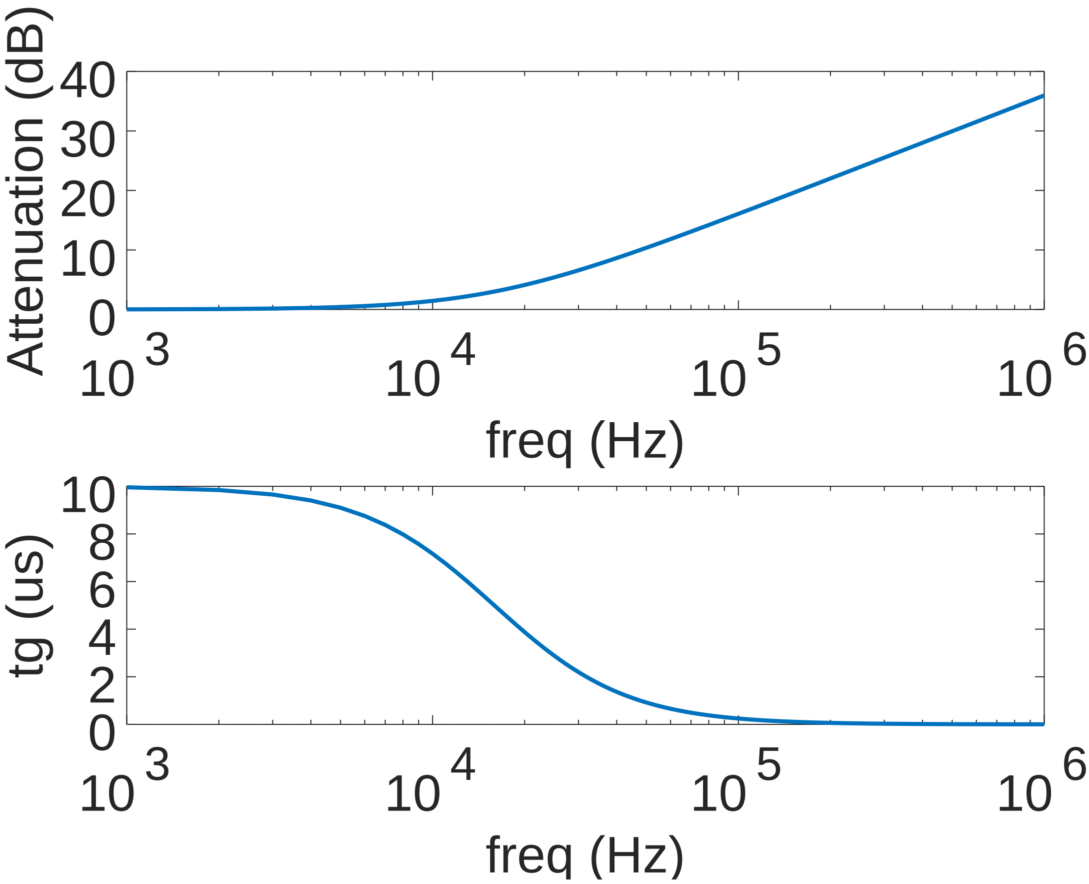

FIGURE 2 – RC filter attenuation and group delay

Réponse 1.4 The group delay and the attenuation of the RC filter are illustrated in Figure 2

Exercice 2 - USB communication filter

The USB Power Delivery (PD) standard was developed to reduce the diversity of needed supplies which can lead to great impact both in terms of cost and enviromental considerations. The USB PD enables a greater functionality of USB power delivery along with data over a single cable. It aims to operate with the existing USB ecosystem. The USB PD standard is based on FSK modulation with a 23.2 MHz carrier frequency and with ±500 kHz deviation. Figure 3 shows a block diagram of a transceiver architecture of a USB PD device.

In this exercise, we will focus our study on the TX filter. The needed filter is a bandpass filter with a useful bandwidth of 1 MHz centered around 23.2 MHz. Its purpose is to remove modulated signal harmonics that arise at different frequencies. For simplicity, we will focus on only one of the constraints which is achieving an attenuation higher than 20 dB for frequencies higher than 26 MHz. An attenuation lower than 1 dB is needed in the useful band.

Question 2.1 Determine the bandpass filter template with a geometric symmetry. Determine the values of f_1, f_2, f_3, f_4, A_{max} and A_{min}.

Réponse 2.1 Determining f_2 and f_3 is straightforward, they are given respectively by 23.2-0.5 and 23.2 + 0.5 in order to have an overall bandwidth of 1 MHz. f_4 is a given specifications it is 26

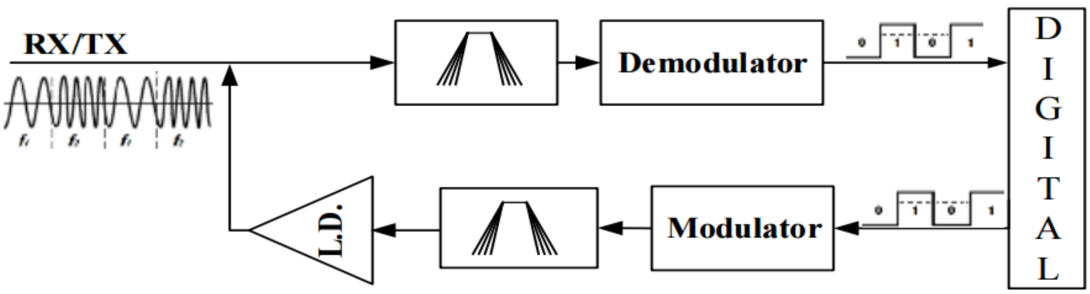

FIGURE 3 – FSK transceiver

MHz. Regarding f_1, since the designed filter need to have a geometric symmetry, we need to satisfy $f_1 f_4 = f_2 f_3 \implies f_1$=20.69 MHz. Regarding the attenuation, its minimum value in the stop band $A_{min} = 20$ dB and its max value in the pass band A_{max}=1 dB.

Question 2.2 Determine the selectivity parameter Ω_S and the low-pass prototype template.

Réponse 2.2 $\Omega_s = \frac{f_4 - f_1}{f_3 - f_2} = \frac{26-20.69}{23.7-22.7} = 5.31$

Question 2.3 Calculate the order of the prototype filter for a polynomial approximation of Butterworth.

Réponse 2.3

$$\Psi_n(\Omega_s) = \Omega_s^n \geq D = \sqrt{\frac{10^{A_{min}/10} - 1}{10^{A_{max}/10} - 1}}$$

$D = 19.48 \implies n \geq \frac{\log(D)}{\log \Omega_s} = 1.76 \implies n = 2$

Question 2.4 Calculate the possible range for ϵ

Réponse 2.4

$$\epsilon_{max} = \sqrt{10^{\frac{Amax}{10}} - 1} = 0.5088$$

$$\epsilon_{min} = \sqrt{\left(\frac{10^{\frac{Amin}{10}} - 1}{\Omega_s^{2n}}\right)} = 0.353$$

Question 2.5 Using the Butterworth table, determine the transfer function of the prototype filter in the Laplace domain for ϵ_{min}.

Réponse 2.5 We have a second order filter, therefore the Normalized [1] transfer function for a Butterworth approximation is given by :

$$T_{NormalizedLowPass}(S_N) = \frac{1}{S_N^2 + 1.414 S_N + 1}$$

We use apply the transformation $S_N \longrightarrow S \cdot \epsilon^{\frac{1}{n}}$ to adapt it to the attenuation constraints

$$T_{LowPass}(S) = \frac{1}{(\epsilon^{\frac{1}{n}} S)^2 + 1.414 \epsilon^{\frac{1}{n}} S + 1} = \frac{1}{0.35 S^2 + 0.84 S + 1}$$

1. Normalized with respect to an in-band attenuation of 3 dB

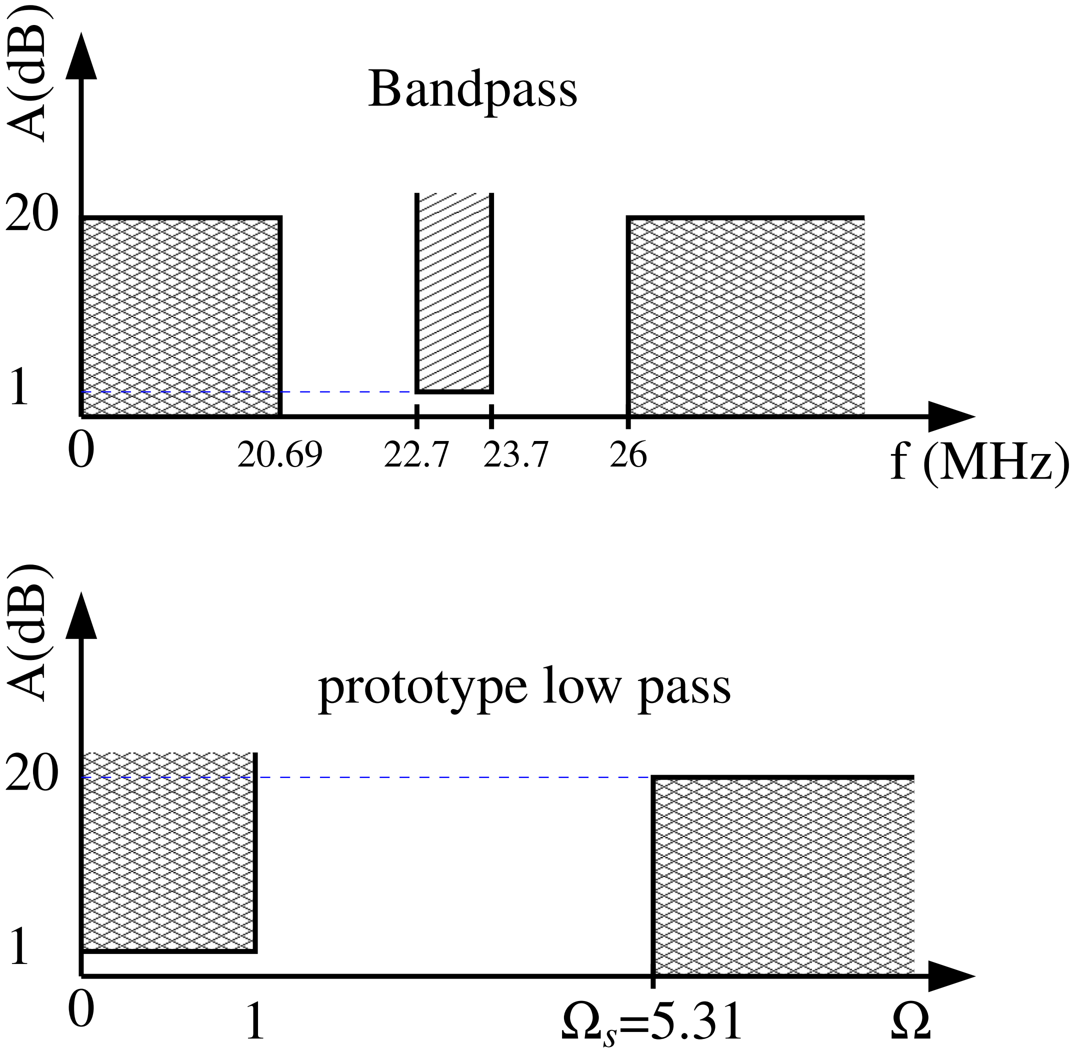

FIGURE 4 – Filter template

Question 2.6 Determine the expression of the equivalent bandpass selection filter.

Réponse 2.6 We just have to apply the following transformation $S \longrightarrow \frac{\omega_o}{2\pi B}\left[\frac{p}{\omega_o} + \frac{\omega_o}{p}\right]$

Exercice 3 - USB bandpass Filter Implementation

To implement the filter of the USB bandpass Receiver, we would like to build it based on the cell presented in Figure 5. As can be noticed, this cell has 3 inputs V_1, V_2 and V_3, and one output V_{out}. The output current of the transconductances is given by $I_= gm_i(V^+ -^V -)$. Moreover all the transconductances have an infinite input impedance which translates into a null current at its inputs.m

Question 3.1 Determine the transfer function of the cell in the Laplace domain $V_{out}(p) = f(V_1(p), V_2(p), V_3(p))$

Order	Numerator	Denominator
2	1	$S_N^2 + 1.414S_N + 1$
3	1	$(S_N + 1)(S_N^2 + S_N + 1)$
4	1	$(S_N^2 + 0.765S_N + 1)(S_N^2 + 1.848S_N + 1)$
5	1	$(S_N + 1)(S_N^2 + 0.618S_N + 1)(S_N^2 + 1.618S_N + 1)$
6	1	$(S_N^2 + 0.518S_N + 1)(S_N^2 + 1.414S_N + 1)(S_N^2 + 1.932S_N + 1)$

TABLE 1 – Normalized Butterworth table

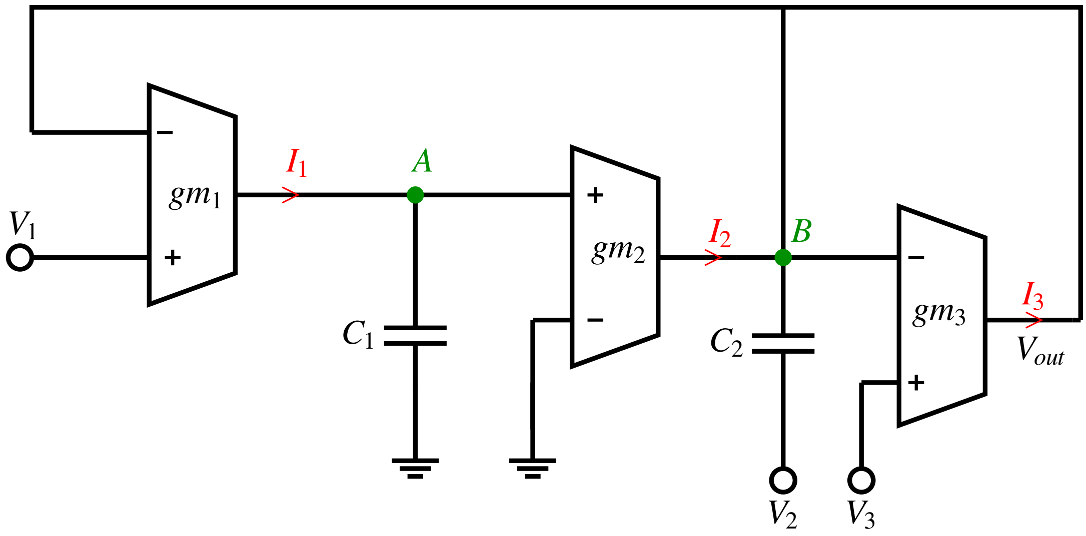

FIGURE 5 – Gm Filter

Réponse 3.1

$$I_1(p) = gm_1(V_1(p) - V_{out}(p))$$

$$V_A(p) = \frac{I_1(p)}{C_1\,p} = \frac{gm_1(V_1(p) - V_{out}(p))}{C_1 p}$$

$$I_2(p) = gm_2\,V_A(p)$$

$$I_3(p) = gm_3(V_3(p) - V_{out}(p))$$

$$V_B(p) - V_2(p) = V_{out}(p) - V_2(p) = \frac{I_2(p) + I_3(p)}{C_2 p}$$

By arranging all the terms, we obtain

$$V_{out}(p) = \frac{gm_1 gm_2 V_1(p) + C_1 C_2 p^2 V_2(p) + gm_3 C_1 p V_3(p)}{C_1 C_2 p^2 + gm_3 C_1 p + gm_1 gm_2}$$

Question 3.2 Plot the Bode diagram of the modulus of the transfer function [2] in the 3 following configurations and determine the filtering performed in each case

- $V_2(p) = 0$ and $V_3(p) = 0$
- $V_1(p) = 0$ and $V_3(p) = 0$
- $V_1(p) = 0$ and $V_2(p) = 0$

2. Do not calculate the poles of the function, assume that it has two poles ω_I and ω_{II}

Réponse 3.2 The transfer function has 2 poles and for

— $V_2(p) = 0$ and $V_3(p) = 0 \Longrightarrow$ it has no zeros, therefore it behaves as a low pass filter when the input is applied at V_1

— $V_1(p) = 0$ and $V_3(p) = 0 \Longrightarrow$ it has two zeros at DC , therefore it behaves as a high pass filter when the input is applied at V_2

— $V_1(p) = 0$ and $V_2(p) = 0 \Longrightarrow$ it has one zero at DC, therefore it behaves as a band pass filter when the input is applied at V_3

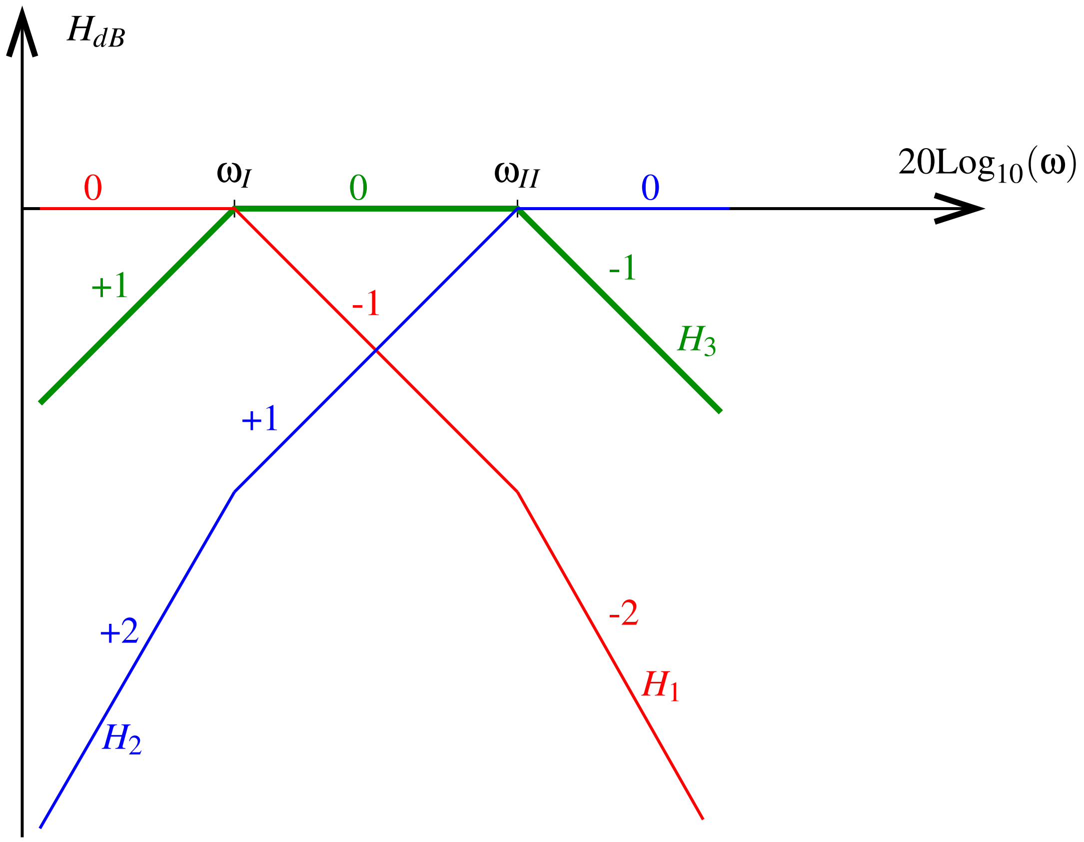

FIGURE 6 – Bode diagram for H_i

Question 3.3 Express the transfer function for ($V_1(p) = 0$ and $V_2(p) = 0$) as follows :

$$H(p) = \frac{V_{out}(p)}{V_3(p)} = \frac{\frac{p}{Q\cdot\omega_0}}{\frac{p^2}{\omega_0^2} + \frac{p}{Q\cdot\omega_0} + 1}$$

Determine the expressions of ω_0 and Q.

Réponse 3.3 For $V_1(p) = 0$ and $V_2(p) = 0$

$$V_{out}(p) = \frac{gm_3C_1pV_3(p)}{C_1C_2p^2 + gm_3C_1p + gm_1gm_2}$$

$$H(p) = \frac{V_{out}(p)}{V_3(p)} = \frac{gm_3C_1p}{C_1C_2p^2 + gm_3C_1p + gm_1gm_2} = \frac{\frac{gm_3C_1}{gm_1gm_2}p}{\frac{C_1C_2}{gm_1gm_2}p^2 + \frac{gm_3C_1}{gm_1gm_2}p + 1}$$

$\omega_0 = \sqrt{\frac{gm_1gm_2}{C_1C_2}}$ and $Q = \sqrt{\frac{gm_1gm_2C_2}{gm_3^2C_1}}$

Question 3.4 By drawing the cell of Figure 5 as a black box with 3 inputs and 1 output, propose a possible implementation of the filter needed for the USB Tx.

Réponse 3.4 We need a fourth order Band Pass filter. The used cell is a second order cell which behaves

- as a low pass filter if the input is applied to V_1
- as a high pass filter if the input is applied to V_2
- as a band pass filter if the input is applied to V_3

Therefore we can implement the fourth order filter by cascading 2 band pass filter cells ($V_3 \longrightarrow V_3$), or one high pass filter cell followed by a low pass filter cell ($V_2 \longrightarrow V_1$), or one low pass filter cell followed by a high pass filter cell ($V_1 \longrightarrow V_2$)