



Elements of Communication Theory & Signals

Slot 4 – I/Q Passband Communication, OFDM and LTE
basics

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Slot 4 – I/Q Passband Communication, OFDM and LTE basics

From a decision variable to a transmitted radio waveform

We know how to shape pulses and decide between noisy symbols. Now we place those symbols on an RF carrier, and ultimately on many orthogonal subcarriers.

1. Construct and recover I/Q waveforms; interpret ASK, PSK and 16-QAM.
2. Relate decision boundaries, E_b/N_0 , BER and EVM in AWGN.
3. Build an OFDM symbol and interpret an LTE downlink resource grid.

Pulse shaping and matched filtering

$$u(t) = \sum_k a_k p(t - kT_{\text{symb}}), \quad (1)$$

$$g(t) = (p * h)(t), \quad (2)$$

$$g(T_0 + \ell T_{\text{symb}}) = \delta(\ell). \quad (3)$$

With correct timing, the decision sample is

$$z_k = a_k + w_k. \quad (4)$$

Now a_k and z_k may be complex: two real coordinates (I and Q).

$$R_{\text{symb}} = 1/T_{\text{symb}}, \quad T_s = 1/f_s. \quad (5)$$

- T_s is a sampling period, not always a symbol period.
- T_0 is the matched-filter delay.
- Use a real unit-energy pulse, $\int |p(t)|^2 dt = 1$, and a unit-gain channel. Thus $p(t)$ is also the pulse arriving at the decision engine, and $E_s = \mathbb{E}[|a_k|^2]$.
- Up to now, we considered a real passband AWGN with two-sided PSD $N_0/2$.
- Symbols are equiprobable.
- Timing, carrier frequency and carrier phase are perfectly known (aka coherent reception).

I/Q: two real waveforms describing one real RF waveform

Define the $\mathbb{C}$ -valued signal:

$$u(t) = \sum_k a_k p(t - kT_{\text{symb}}) = I(t) + jQ(t), \quad j^2 = -1. \quad (6)$$

Call it the **complex envelope**.

We define the $\mathbb{R}$ -valued signal:

$$s(t) = \sqrt{2} \Re\{u(t)e^{j2\pi f_c t}\} \quad (7)$$

$$= \sqrt{2} I(t) \cos(2\pi f_c t) - \sqrt{2} Q(t) \sin(2\pi f_c t). \quad (8)$$

I: in-phase component.

It multiplies the cosine carrier.

$u(t)$ is the baseband equivalent

representation of the real RF signal $s(t)$.

Q: quadrature component.

It multiplies the negative sine carrier.

Write

$$u(t) = \rho(t)e^{j\phi(t)}, \quad (9)$$

$$\rho(t) = \sqrt{I^2(t) + Q^2(t)}, \quad (10)$$

$$\phi(t) = \text{atan2}(Q(t), I(t)) = \arg\{u(t)\}. \quad (11)$$

Therefore

$$s(t) = \sqrt{2}\rho(t)\cos(2\pi f_c t + \phi(t)). \quad (12)$$

Where $\rho(t) > 0$ and phase is differentiable.

Recall the instantaneous frequency:

$$f_i(t) = f_c + \frac{1}{2\pi} \frac{d\phi(t)}{dt}. \quad (13)$$

Connection to previous slots

AM changes the envelope; PM changes phase; FM controls the phase slope.
I/Q can represent all three.

A frequency shift using I/Q

For $u(t) = Ae^{j2\pi f_m t}$,

$$s(t) = \sqrt{2}A\cos(2\pi(f_c + f_m)t). \quad (14)$$

Note that $s(t) = \frac{1}{\sqrt{2}} \left[u(t)e^{j2\pi f_c t} + (u(t)e^{j2\pi f_c t})^* \right]$.

$$S(f) = \frac{1}{\sqrt{2}} [U(f - f_c) + U^*(-f - f_c)]. \quad (15)$$

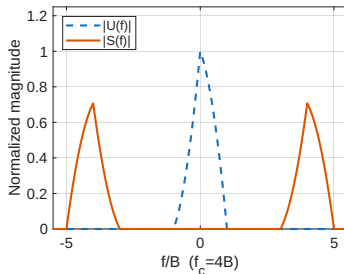
If $U(f) = 0$ for $|f| > B$ and $f_c > B$, the RF band around $+f_c$ has width $2B$.

$$\int s^2(t) dt = \int |u(t)|^2 dt. \quad (16)$$

For root-raised-cosine (RRC) shaping,

$$B = \frac{1 + \alpha}{2} R_{\text{symb}}, \quad B_{\text{RF}} = (1 + \alpha) R_{\text{symb}}. \quad (17)$$

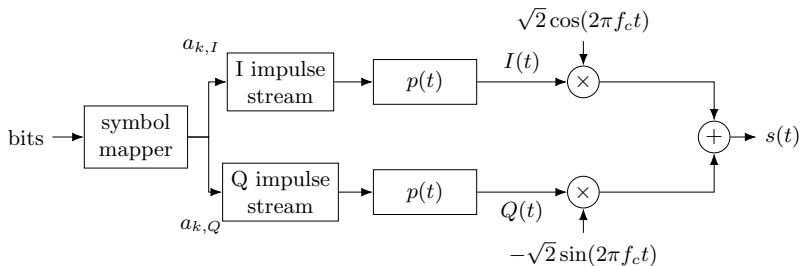
I and Q share this RF band; their bandwidths are not added again.



$u(t)$ is complex, so $U(f)$ has no Hermitian symmetry. The real $s(t)$ must have it.

An ideal I/Q transmitter: mapping, shaping, upconversion

$$a_k = a_{k,I} + ja_{k,Q}, \quad I(t) = \sum_k a_{k,I} p(t - kT_{\text{symb}}), \quad Q(t) = \sum_k a_{k,Q} p(t - kT_{\text{symb}}). \quad (18)$$



Digital implementation: upsample each coordinate sequence, apply a sampled pulse-shaping filter, and reconstruct $I(t)$ and $Q(t)$ before upconversion.

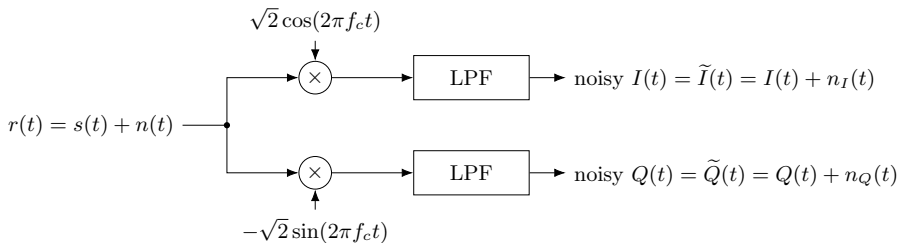
The circles denote ideal multiplication and addition (circuit implementations may slightly differ). A common phase reference generates both carriers.

An ideal coherent I/Q receiver: separate the coordinates

For the noiseless signal, ideal low-pass filtering gives

$$\text{LPF}\{\sqrt{2} s(t) \cos(2\pi f_c t)\} = I(t), \quad (19)$$

$$\text{LPF}\{-\sqrt{2} s(t) \sin(2\pi f_c t)\} = Q(t). \quad (20)$$



Matched filtering and sampling follow on both branches.

With a receiver phase error θ , the recovered envelope is $u(t)e^{-j\theta}$: the constellation rotates. Coherent reception requires phase alignment.

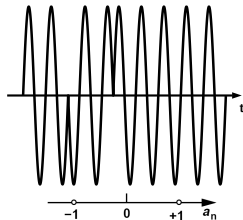
Signal constellation concept and waveforms (♣)

Simple example : Binary PSK and ASK

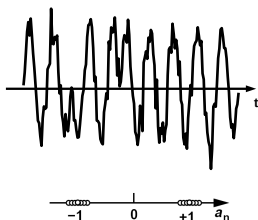
$$x_{PSK}(t) = a_n \cos \omega_c t, \quad a_n = \pm 1 \quad (21)$$

$$x_{ASK}(t) = a_n \cos \omega_c t, \quad a_n = \{0, 1\} \quad (22)$$

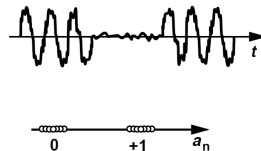
■ Ideal



■ Noisy



■ Noise corrupts the amplitude for both ZEROS and ONES.



- The **constellation** is the graphical representation of the symbols
 - When the transmission is noisy, the received symbols form cloud points around the ideal position
- When the transition between symbols is *continuous*, the constellation points are connected by continuous trajectories in the signal space.

ASK and PSK: choose amplitudes or phases

ASK: use real amplitudes

On-off keying: $a \in \{0, A\}$. (23)

BPSK: use two opposite phases.

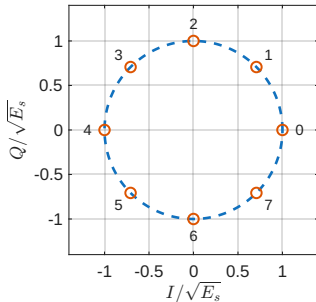
$$a = (2B - 1)\sqrt{E_b}, \quad B \in \{0, 1\}. \quad (24)$$

A negative amplitude means a π phase reversal; it is not negative RF power.

General case M -PSK: equally spaced points on a circle.

$$a = \sqrt{E_s} e^{j(\phi_0 + 2\pi i/M)}, \quad i = 0, \dots, M-1. \quad (25)$$

Examples: BPSK, QPSK and 8-PSK.



QPSK: two antipodal bit streams in one RF band

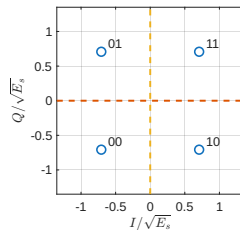
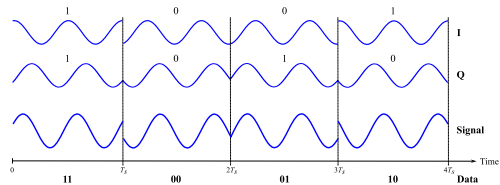
Group bits into $(b_I, b_Q) \in \{0, 1\}^2$ and map

$$a = \sqrt{\frac{E_s}{2}} [(2b_I - 1) + j(2b_Q - 1)]. \quad (26)$$

Each axis carries one bit, with energy $E_b = E_s/2$.

Bits $b_I b_Q$	$a/\sqrt{E_s}$
00	$(-1 - j)/\sqrt{2}$
01	$(-1 + j)/\sqrt{2}$
11	$(1 + j)/\sqrt{2}$
10	$(1 - j)/\sqrt{2}$

Adjacent points differ in one bit: **Gray labeling**.



QPSK and 4-QAM are identical in terms of their constellation geometry.

16-QAM: two independent four-level PAM coordinates

For equally likely symbols,

$$a = a_I + ja_Q, \quad a_I, a_Q \in \{-3d, -d, d, 3d\}. \quad (27)$$

The average symbol energy is

$$E_s = \sum_{a_I, a_Q \in \{-3d, -d, d, 3d\}} \frac{|a_I|^2 + |a_Q|^2}{16} = 10d^2, \quad (28)$$

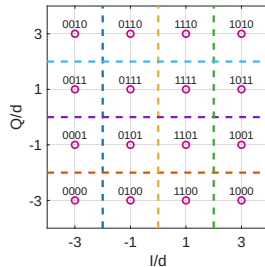
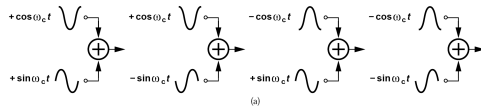
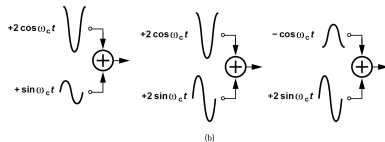
$$d = \sqrt{E_s/10}, \quad E_b = E_s/4. \quad (29)$$

Use the Slot 3 Gray labels on each axis:

level/ d	-3	-1	1	3
label	00	01	11	10

(30)

Thresholds on each coordinate: $-2d, 0, 2d$.



Let $q = \log_2 M$ denote bits per symbol; reserve k for the symbol index.

$$R_b = qR_{\text{sympb}}, \quad E_s = qE_b, \quad \eta = \frac{R_b}{B_{\text{RF}}} = \frac{q}{1 + \alpha}. \quad (31)$$

These are single-carrier results with RRC shaping (and no overhead).

Modulation	q	$d_{\min}$	$d_{\min}^2/E_s$	R_b at $R_{\text{sympb}} = 1$ Msymbol/s
BPSK	1	$2\sqrt{E_s}$	4	1 Mbit/s
QPSK	2	$\sqrt{2E_s}$	2	2 Mbit/s
16-QAM	4	$2d$	2/5	4 Mbit/s

$$d_{\min} = \min_{a \neq a' \in \mathcal{A}} |a - a'|. \quad (32)$$

With $\alpha = 0.25$, all three occupy 1.25 MHz at this symbol rate.

The trade-off

At fixed E_s , more densely packed points have less noise margin.

AWGN in the I/Q plane: use two scalar Gaussian models

After unit-energy matched filtering,

$$z_k = a_k + (w_{k,I} + jw_{k,Q}), \quad (33)$$

$$w_{k,I}, w_{k,Q} \sim \mathcal{N}(0, N_0/2), \quad (34)$$

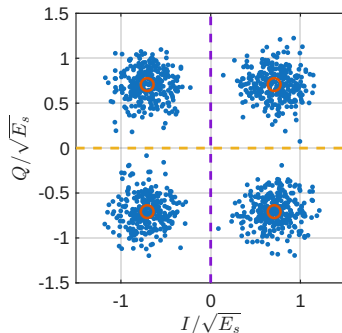
$$\mathbb{E}[|w_k|^2] = N_0. \quad (35)$$

The two real noise coordinates are independent. No two-dimensional probability density is needed.

For equal priors, select the closest point:

$$\hat{a}_k = \arg \min_{a \in \mathcal{A}} |z_k - a|^2. \quad (36)$$

For QPSK and square QAM, this separates into two scalar decisions.



Noise vs Error

Noise moves a point; an error occurs only when a decision boundary is crossed.

The main result survives RF upconversion: coherent BPSK BER

For $B = 0 \mapsto -\sqrt{E_b}$ and $B = 1 \mapsto +\sqrt{E_b}$, the useful observation is

$$Y = \Re\{z\} = (2B - 1)\sqrt{E_b} + N, \quad N \sim \mathcal{N}(0, N_0/2). \quad (37)$$

Equal priors give midpoint threshold $\gamma = 0$: decide 1 for $Y > 0$.

$$P(\hat{B} = 1 \mid B = 0) = P(N > \sqrt{E_b}) = Q\left(\sqrt{2E_b/N_0}\right), \quad (38)$$

$$P(\hat{B} = 0 \mid B = 1) = P(N < -\sqrt{E_b}) = Q\left(\sqrt{2E_b/N_0}\right). \quad (39)$$

Thus, using total probability,

$$P_b = \frac{1}{2}P(\text{error} \mid B = 0) + \frac{1}{2}P(\text{error} \mid B = 1) = Q\left(\sqrt{2E_b/N_0}\right). \quad (40)$$

The Q branch contains noise but no BPSK information. At $Y = 0$, either decision has the same conditional error; this event has probability zero.

For the Gray mapping of slide 11,

$$Y_I = (2b_I - 1)\sqrt{E_b} + N_I, \quad (41)$$

$$Y_Q = (2b_Q - 1)\sqrt{E_b} + N_Q, \quad (42)$$

where $E_s = 2E_b$ and N_I, N_Q are independent $\mathcal{N}(0, N_0/2)$.

Both decision thresholds are zero. Each bit has

$$P_b = Q\left(\sqrt{2E_b/N_0}\right) = Q\left(\sqrt{E_s/N_0}\right). \quad (43)$$

QPSK carries two bits per symbol without changing the bit error probability at the same E_b/N_0 .

A symbol is correct only if both bits are correct:

$$P_s = 1 - (1 - P_b)^2 \quad (44)$$

$$= 2P_b - P_b^2. \quad (45)$$

$P_s \simeq 2P_b$ is only a low-error-rate approximation.

16-QAM: reuse the scalar four-PAM error calculation

Let $\sigma^2 = N_0/2$. Each coordinate is four-PAM:

$$p_{\text{axis}} = \frac{3}{2}Q\left(\frac{d}{\sigma}\right), \quad \frac{d}{\sigma} = \sqrt{\frac{E_s}{5N_0}}. \quad (46)$$

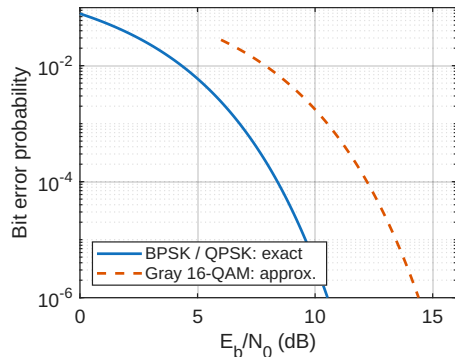
Independent coordinates give the exact SER:

$$P_s = 1 - \left[1 - \frac{3}{2}Q\left(\sqrt{\frac{E_s}{5N_0}}\right)\right]^2. \quad (47)$$

For Gray labels at high SNR,

$$P_b \approx \frac{3}{4}Q\left(\sqrt{\frac{4E_b}{5N_0}}\right). \quad (48)$$

The BER approximation is not exact at low SNR.



- Requires further assumptions (nearest neighbor approximation)

Modulation	$P_s(\gamma_s)$	$P_b(\gamma_b)$
BPSK		$= Q(2\sqrt{\gamma_b})$
QPSK	$\approx 2Q(\sqrt{\gamma_s})$	$\approx Q(2\sqrt{\gamma_b})$
MPSK	$\approx 2Q(\sqrt{2\gamma_s} \sin(\frac{\pi}{M}))$	$\approx \frac{2}{\log_2 M} Q(\sqrt{2\gamma_b \log_2 M} \sin(\frac{\pi}{M}))$
M-QAM	$\approx 4Q(\sqrt{\frac{3\gamma_s}{M-1}})$	$\approx \frac{4}{\log_2 M} Q(\sqrt{\frac{3\gamma_b \log_2 M}{M-1}})$

$$\gamma_s = E_s/N_0 \quad (49)$$

$$\gamma_b = E_b/N_0 \quad (50)$$

$$\gamma_b = \frac{\gamma_s}{\log_2 M} \quad (51)$$

$$P_b \approx \frac{P_s}{\log_2 M} \quad (52)$$

- Common form :

$$P_s(\gamma_s) \approx \alpha_M Q(\sqrt{\beta_M \gamma_s}) \quad (53)$$

$$P_b(\gamma_b) \approx \hat{\alpha}_M Q(\sqrt{\hat{\beta}_M \gamma_b}) \quad (54)$$

Refer to [1] for more details on error performance in digital communication systems.

Error vector magnitude: measure distance from the reference

For known reference symbols a_k and aligned received samples z_k , define

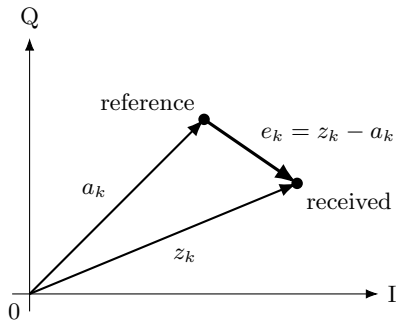
$$e_k = z_k - a_k. \quad (55)$$

The average-reference-power normalized RMS EVM is

$$\text{EVM}_{\text{rms}} = \sqrt{\frac{\sum_{k=1}^K |e_k|^2}{\sum_{k=1}^K |a_k|^2}}. \quad (56)$$

$$\text{EVM}_{\%} = 100 \text{EVM}_{\text{rms}}. \quad (57)$$

EVM requires a reference sequence. Timing, gain, phase correction and normalization must be applied before computing EVM.



For aligned, gain-corrected symbols with **AWGN only**, define

$$\text{SNR}_{\text{sym}} = \frac{E_s}{N_0}. \quad (58)$$

The theoretical normalized mean-square error is

$$\frac{\mathbb{E}[|z - a|^2]}{\mathbb{E}[|a|^2]} = \frac{N_0}{E_s}. \quad (59)$$

For a sufficiently long measurement,

$$\text{EVM}_{\text{rms}} \approx \frac{1}{\sqrt{\text{SNR}_{\text{sym}}}}, \quad (60)$$

$$\text{EVM}_{\text{dB}} \approx -\text{SNR}_{\text{sym,dB}}. \quad (61)$$

Here $\text{EVM}_{\text{dB}} = 20 \log_{10}(\text{EVM}_{\text{rms}})$.

AWGN example

$\text{EVM} = 10\%$ gives $E_s/N_0 \approx 100 = 20 \text{ dB}$.
For QPSK, $E_b/N_0 \approx 50$.

EVM alone does not determine BER.

BER also depends on the constellation, labeling, decision regions and error distribution.

For example, uncompensated phase error gives

$$z = ae^{j\theta}, \quad \text{EVM} = 2|\sin(\theta/2)|, \quad (62)$$

which is not an AWGN cloud.

OFDM: transmit many QAM symbols in parallel

Single carrier: one stream of symbols multiplying shifted pulses.

$$u(t) = \sum_k a_k p(t - kT_{\text{symb}}). \quad (63)$$

OFDM: within block ℓ , one symbol on each active subcarrier m .

$$u_\ell(t) = \sum_{m \in \mathcal{K}} a_{m,\ell} \cdot \psi_m(t), \quad \psi_m(t) = \frac{1}{\sqrt{T_u}} e^{j2\pi m \Delta_F t}, \quad 0 \leq t < T_u. \quad (64)$$

$\mathcal{K}$ is the active-subcarrier set; T_u is the useful OFDM duration.

Each $a_{m,\ell}$ is a $\mathbb{C}$ symbol. The *sum* $u_\ell(t)$ is a superposition of many complex symbols.

Why organize symbols this way?

Parallel transmission permits a long useful block and flexible frequency allocation. Orthogonal subcarriers can overlap spectrally yet be separated by the receiver.

Orthogonality: overlapping spectra, zero cross-projection

Choose the frequency spacing for orthogonal subcarriers:

$$\Delta_F = 1/T_u. \quad (65)$$

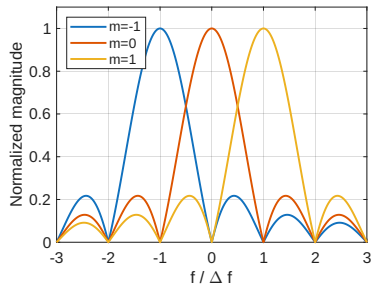
For integer subcarrier indices,

$$\int_0^{T_u} \psi_m(t) \psi_n^*(t) dt = \frac{1}{T_u} \int_0^{T_u} e^{j2\pi(m-n)t/T_u} dt \quad (66)$$

$$= \begin{cases} 1, & m = n, \\ 0, & m \neq n. \end{cases} \quad (67)$$

Hence:

$$\int_0^{T_u} u_\ell(t) \psi_n^*(t) dt = \sum_{m \in \mathcal{K}} a_{m,\ell} \underbrace{\int_0^{T_u} \psi_m(t) \psi_n^*(t) dt}_{\substack{1 \text{ if } m=n, \\ 0 \text{ otherwise}}} = a_{n,\ell}. \quad (68)$$



The integration interval is T_u , not the cyclic-prefix-extended duration.

The IFFT builds the (time-domain) waveform; the FFT recovers its symbols

Use an even N and signed bins $m = -N/2, \dots, N/2 - 1$; unused $a_{m,\ell}$ are zero.

$$f_s = N\Delta_F, \quad T_s = 1/f_s, \quad T_u = NT_s. \quad (69)$$

Define *energy-normalized* samples $x_\ell[n] = \sqrt{T_s} u_\ell(nT_s)$:

$$x_\ell[n] = \frac{1}{\sqrt{N}} \sum_m a_{m,\ell} e^{j2\pi mn/N}, \quad n = 0, \dots, N-1, \quad (70)$$

$$a_{m,\ell} = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x_\ell[n] e^{-j2\pi mn/N}. \quad (71)$$

With this **unitary** normalization, Parseval gives

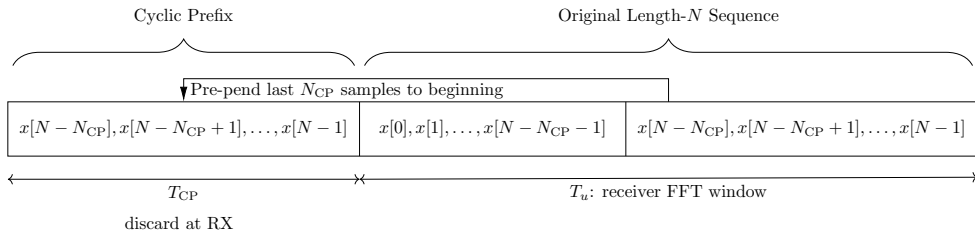
$$\sum_n |x_\ell[n]|^2 = \sum_m |a_{m,\ell}|^2. \quad (72)$$

Matlab

```
N = 10; A = complex((randi(4,N,1)-2.5)*2,(randi(4,N,1)-2.5)*2)/sqrt(2);  
x = sqrt(N)*ifft(ifftshift(A));  
Recover with Arx = fftshift(fft(x)/sqrt(N));
```

The cyclic prefix: repeat the end before sending the block

Copy the last N_{CP} samples of the useful block to its beginning.

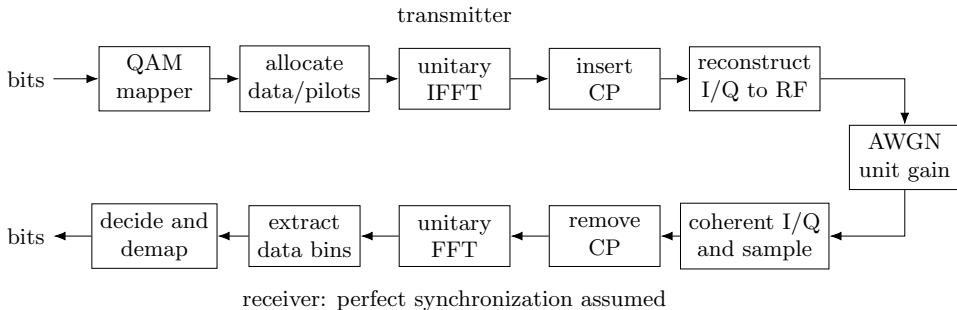


$$T_{CP} = N_{CP} T_s, \quad T_{OFDM} = T_u + T_{CP}, \quad \eta_{\text{time}} = \frac{T_u}{T_u + T_{CP}}. \quad (73)$$

Purpose: accommodate delayed copies of a signal while preserving the useful FFT interval, provided the prefix covers the relevant delay spread and timing margin.

That channel analysis is outside this slot. In the memoryless AWGN model, the prefix is unnecessary for detection: it consumes time and energy, and does not add new data or an AWGN gain.

An OFDM transceiver still uses one I/Q upconverter



The IFFT replaces a bank of individually generated subcarriers. Its real and imaginary outputs feed the same I/Q architecture as before.

At RX, synchronization identifies the useful block. The FFT separates subcarriers; each data bin then uses its own QPSK or QAM decision rule.

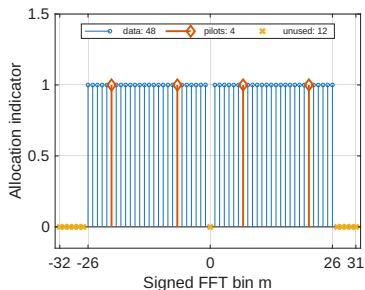
Teaching example, not a true LTE allocation:

$$N = 64, \quad \mathcal{K} = \{-26, \dots, -1, 1, \dots, 26\}. \quad (74)$$

Four pilot bins: $m \in \{-21, -7, 7, 21\}$.

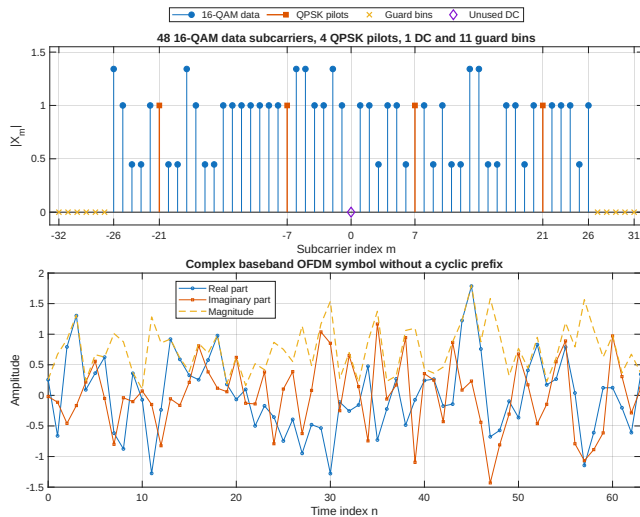
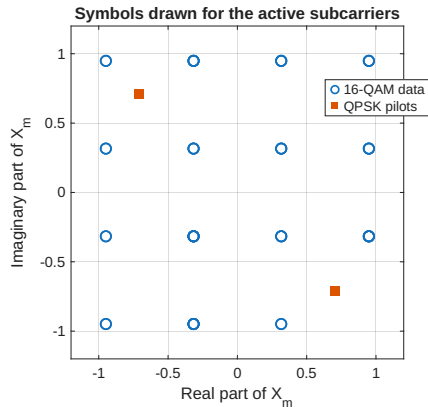
There are 48 data bins, 4 known pilot bins, 1 unused DC bin and 11 edge guard bins.

Pilots provide known references for estimating gain, phase or timing. Guard bins leave room near the sampled-band edges; they do not eliminate sidelobes.



Do **not** impose conjugate symmetry: wireless I/Q baseband is generally complex. The later RF conversion produces a real waveform.

Example continued



AWGN after the FFT: the same noise energy per data symbol

After prefix removal, assume the ideal energy-normalized discrete model

$$y[n] = x[n] + w[n], \quad w[n] = w_I[n] + jw_Q[n], \quad w_I[n], w_Q[n] \sim \mathcal{N}(0, N_0/2). \quad (75)$$

All real noise samples are independent. On subcarrier m ,

$$Z_m = a_m + W_m, \quad W_m = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} w[n] e^{-j2\pi mn/N}. \quad (76)$$

Independence and zero means eliminate cross terms, so

$$\mathbb{E}[|W_m|^2] = \frac{1}{N} \sum_{n=0}^{N-1} \mathbb{E}[|w[n]|^2] = N_0. \quad (77)$$

The real and imaginary coordinates remain independent Gaussians with variance $N_0/2$.

No intrinsic AWGN penalty or gain

At the same *data-symbol* E_s/N_0 , QPSK and QAM have the same decision boundaries and BER as their single-carrier versions.

OFDM: data-bit rate and energy per data symbol

Uncoded M -ary modulation: $q = \log_2 M$ bits per data symbol on N_d data subcarriers. Assume equiprobable symbols, unit channel gain and ideal coherent reception in AWGN.

Data-bit rate

$$R_b = \frac{N_d q}{T_u + T_{CP}}. \quad (78)$$

Each OFDM block carries $N_d q$ new data bits.

Cyclic prefix: repeats samples and increases transmission time, without adding new bits.

Pilots: carry known reference symbols, not data bits. (N_p pilot subcarriers per OFDM block.)

Unused bins: their coefficients are zero. They carry neither data nor pilot symbols. Ideally, their energy is zero, but they lock the corresponding resource in frequency.

Energy per data-subcarrier symbol

Using the unit-energy subcarrier basis over T_u ,

$$E_s = \mathbb{E}[|a_{m,\ell}|^2]. \quad (79)$$

The energy per mapped bit is $E_b = E_s/q$.

Neither quantity includes CP or pilot energy.

Gray QPSK on each data subcarrier

$$P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right). \quad (80)$$

At fixed E_s/N_0 per data subcarrier, this is the single-carrier QPSK result.

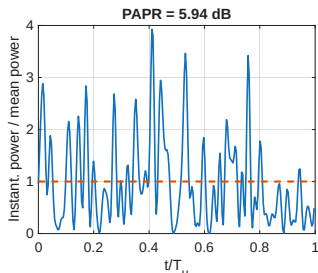
Two OFDM cautions: waveform peaks and synchronization

PAPR

Subcarrier contributions sometimes add in phase, producing large peaks. Over a useful block, define

$$\text{PAPR} = \frac{\max_{0 \leq t < T_u} |u(t)|^2}{\frac{1}{T_u} \int_0^{T_u} |u(t)|^2 dt}. \quad (81)$$

Amplitude clipping changes the signal, increasing EVM and out-of-band energy.



CFO sensitivity

A carrier-frequency offset inserts $e^{j2\pi\delta_F t}$ into the received waveform.

$$u_{rx}(t) = u(t)e^{j2\pi\delta_F t} \\ \Rightarrow u_{rx}[n] = u[n]e^{j2\pi\delta_F n T_s} \quad (82)$$

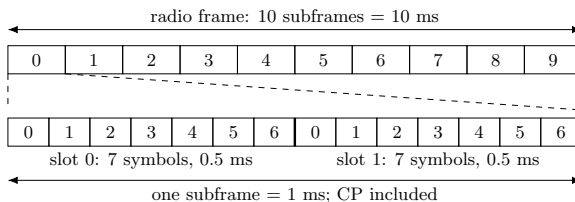
The exact orthogonality integral then fails, causing intercarrier interference.

Time synchronization sensitivity

Obviously, the receiver must be synchronized in time to correctly capture the OFDM symbols. Timing errors shift the received waveform relative to the receiver's FFT window, also causing intercarrier interference.

$$u_{rx}(t) = u(t - \delta_t) \\ \Rightarrow u_{rx}[n] = u[n - \delta_n] \quad (83)$$

LTE example: Release 8 downlink, FDD, normal cyclic prefix



$$\Delta_F = 15 \text{ kHz}, \quad T_u = 66.667 \mu\text{s}. \quad (84)$$

LTE reference time unit:

$$\tau_{\text{LTE}} = \frac{1}{2048 \times 15\,000} \text{ s}. \quad (85)$$

CP in *each slot*: $160\tau_{\text{LTE}} \simeq 5.208 \mu\text{s}$ for symbol 0; $144\tau_{\text{LTE}} = 4.6875 \mu\text{s}$ for symbols 1–6.

Downlink: OFDM waveform; OFDMA assigns different frequency/time resources to users.

Uplink: LTE uses DFT-spread OFDM (SC-FDMA); it is not the same plain OFDM chain.

We use one antenna port and QPSK/16-QAM examples. LTE also includes coding, synchronization and reference signals.

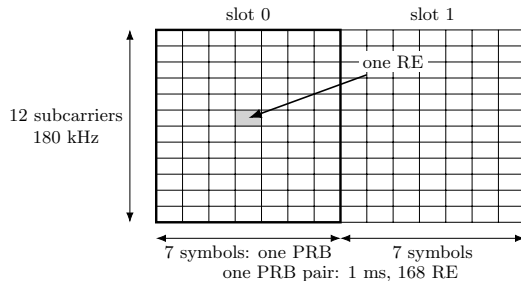
A **resource element (RE)** is one subcarrier during one OFDM symbol, for one antenna port.

With normal CP, a **physical resource block (PRB)** spans

$$12 \text{ subcarriers} \times 7 \text{ symbols} = 84 \text{ RE. (86)}$$

Its width is $12 \times 15 = 180 \text{ kHz}$ and its duration is one 0.5 ms slot.

A pair across a subframe contains 168 RE, before subtracting non-data resources.



REs may carry data, control, reference or synchronization signals. The drawing shows dimensions, not their detailed LTE placement.

LTE channel bandwidth is not the number of FFT bins

Channel (MHz)	N_{RB}	Active bins
1.4	6	72
3	15	180
5	25	300
10	50	600
15	75	900
20	100	1200

Active bins: $12N_{\text{RB}}$. The downlink DC bin is not transmitted; remaining edge bins can be zero.

Example implementation for a 10 MHz channel

$$N = 1024, \quad f_s = N\Delta_F = 15.36 \text{ MHz}, \quad (87)$$

$$\mathcal{K} = \{-300, \dots, -1, 1, \dots, 300\}. \quad (88)$$

The CP is 80 samples for the first symbol of each slot, then 72 samples.

$600 \times 15 \text{ kHz} = 9 \text{ MHz}$ is the *sum of allocated subcarrier widths*, not a strict spectral-support width: account also for the DC gap, guard regions and sidelobes.

FFT size is an implementation choice; it is not the RF channel bandwidth expressed in bins.

Exercise – From QPSK decisions to an OFDM resource grid

Assume equiprobable bits, the slide 11 Gray mapping, unit channel gain, perfect synchronization and AWGN. Use energy units such that $E_s = 1$. A long QPSK measurement gives $\text{RMS EVM} = 25\%$;

- (a) Mapping and decisions.
Consider $(b_I, b_Q) = (0, 1)$. Write its isolated-pulse RF waveform using a real, unit-energy transmitter pulse $p(t)$, and give the two optimal thresholds.
- (b) EVM and BER.
Find N_0 , each real noise standard deviation, E_b/N_0 and the exact QPSK BER.
- (c) OFDM timing and rate.
Use $N = 64$, $\Delta_F = 15$ kHz, $N_{\text{CP}} = 16$, $N_d = 48$ and $N_p = 4$. Find f_s , T_u , T_{CP} and the data-bit rate.
- (d) FFT noise and error rate.
Keep the same E_s and N_0 as in (b). After CP removal, the independent AWGN samples satisfy $\mathbb{E}[w[n]] = 0$ and $\mathbb{E}[|w[n]|^2] = N_0$. Using a unitary FFT, find the output noise mean-square magnitude and the variance of each real noise coordinate. Deduce the BER on each QPSK data subcarrier.
- (e) LTE counting.
For a 10 MHz LTE downlink with 50 RB, normal CP and one antenna port, count REs in one subframe. Give the QPSK bit-rate upper bound if every RE carried data. Why is it not a payload-throughput prediction?
Use $Q(x) = \frac{1}{2} \operatorname{erfc}(x/\sqrt{2})$.

1. Map the bits to their corresponding QPSK constellation point.

With $E_s = 1$,

$$a_I = \frac{2(0) - 1}{\sqrt{2}} = -\frac{1}{\sqrt{2}}, \quad a_Q = \frac{2(1) - 1}{\sqrt{2}} = \frac{1}{\sqrt{2}}, \quad \boxed{a = \frac{-1 + j}{\sqrt{2}}}. \quad (89)$$

2. Form the complex baseband pulse and upconvert to RF.

For a real unit-energy pulse $p(t)$,

$$u(t) = \frac{-1 + j}{\sqrt{2}} p(t), \quad (90)$$

$$s(t) = \sqrt{2} \left[-\frac{p(t)}{\sqrt{2}} \cos(2\pi f_c t) - \frac{p(t)}{\sqrt{2}} \sin(2\pi f_c t) \right] \quad (91)$$

$$= \boxed{-p(t) \cos(2\pi f_c t) - p(t) \sin(2\pi f_c t)}. \quad (92)$$

3. Place the midpoint thresholds for I and Q axes.

Each axis has equiprobable levels $\pm 1/\sqrt{2}$ with equal Gaussian noise variance, so

$$\boxed{\gamma_I = \gamma_Q = 0}, \quad \hat{b}_I = 1_{\{\Re z > 0\}}, \quad \hat{b}_Q = 1_{\{\Im z > 0\}}. \quad (93)$$

1. Convert EVM to noise energy and standard deviation.

$$\text{EVM}^2 = \frac{N_0}{E_s} = 0.25^2 = 0.0625 \xrightarrow{E_s=1} N_0 = 0.0625, \quad \sigma = \sqrt{N_0/2} = \sqrt{\frac{0.25^2}{2}} \approx 0.1768. \quad (94)$$

2. Convert symbol energy to bit energy.

$$E_b = E_s/2 = \frac{1}{2}, \quad \boxed{E_b/N_0 = 8 \approx 9 \text{ dB.}} \quad (95)$$

$$P_e(E_b/N_0) = Q(\sqrt{2E_b/N_0}) = Q(4) \approx 3.17 \times 10^{-5}. \quad (96)$$

1. Determine the sampling rate and durations.

$$f_s = N\Delta_F = 960 \text{ kSamples/s}, \quad (97)$$

$$T_u = \frac{1}{\Delta_F} = 66.667 \text{ } \mu\text{s}, \quad (98)$$

$$T_{\text{CP}} = \frac{N_{\text{CP}}}{f_s} = 16.667 \text{ } \mu\text{s}, \quad (99)$$

$$T_{\text{OFDM}} = T_u + T_{\text{CP}} = 83.333 \text{ } \mu\text{s}. \quad (100)$$

2. Data bit-rate.

QPSK carries $q = 2$ bits per data-subcarrier symbol:

$$R_b = \frac{N_d \times \log_2(M)}{T_{\text{OFDM}}} = \frac{48 \times 2}{83.333 \text{ } \mu\text{s}} \approx 1.152 \text{ Mbit/s}. \quad (101)$$

Pilots carry no new data bits. The CP repeats samples and is included in the block duration.

1. Apply the unitary FFT to the noise.

On data subcarrier m , $Z_m = a_m + W_m$, with

$$W_m = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} w[n] e^{-j2\pi mn/N}. \quad (102)$$

2. Compute the output noise statistics.

Independence and zero means eliminate cross terms:

(use $|W_m|^2 = W_m \times W_m^*$ and $|w[n]|^2 = w_I[n]^2 + w_Q[n]^2$ (Rayleigh distribution))

$$\mathbb{E}[|W_m|^2] = \frac{1}{N} \sum_{n=0}^{N-1} \mathbb{E}[|w[n]|^2] = N_0 = 0.25^2. \quad (103)$$

The real coordinates remain independent Gaussians:

$$\text{Var}(\Re W_m) = \text{Var}(\Im W_m) = \frac{N_0}{2} = \frac{1}{32}. \quad (104)$$

3. Apply the QPSK BER formula.

Since $E_s/N_0 = 16$ and $E_b = E_s/2$,

$$P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q(4) \quad (105)$$

The BER is unchanged at the same data-symbol E_s/N_0 .

Solution (e) - Count the grid, then distinguish raw bits from payload

1. Count subcarriers and OFDM symbols.

$$N_{\text{active}} = 50 \times 12 = 600 \text{ [subcarr]}, N_{\text{sym/subframe}} = 2 \times 7 = 14 \text{ [OFDM symb/subframe]}. \quad (106)$$

2. Resource elements for one subframe and theoretical QPSK data bits.

$$N_{\text{RE}} = 600 \times 14 = 8400 \text{ [REs]}, N_{\text{bits}} = \log_2(4) \times 8400 = 16\,800 \text{ [bits]}. \quad (107)$$

3. Data rate for one subframe.

$$R_{b,\text{upper}} = \frac{N_{\text{bits}}}{T_{\text{subframe}}} = \frac{16\,800}{1 \text{ ms}} = 16.8 \text{ Mbit/s}. \quad (108)$$

Actual payload is lower because some REs are not data, coding adds redundancy, and higher-layer overhead also consumes capacity.



A. Goldsmith, *Wireless Communications*.
Cambridge University Press, 2005.

Appendix slides

Choose the matched filter and retain the overall pulse notation:

$$h(t) = p^*(T_0 - t), \quad g(t) = (p * h)(t). \quad (109)$$

At $t = kT_{\text{sympb}} + T_0$,

$$z_k = a_k g(T_0) + \sum_{\ell \neq k} a_\ell g(T_0 + (k - \ell)T_{\text{sympb}}) + w_k. \quad (110)$$

For a unit-energy root-Nyquist pulse,

$$g(T_0) = 1, \quad g(T_0 + nT_{\text{sympb}}) = 0 \quad (n \neq 0). \quad (111)$$

Hence $z_k = a_k + w_k$: no ISI at the decision instants.

Practical example: RRC at TX and matched RRC at RX produce an overall raised-cosine pulse, after delay alignment.

A constellation describes symbol coordinates, not every point traversed by the continuous waveform $I(t) + jQ(t)$.