



IP PARIS



# Elements of Communication Theory & Signals

Slot 1 – Signals, Spectrum, Sampling and Analog  
Modulation

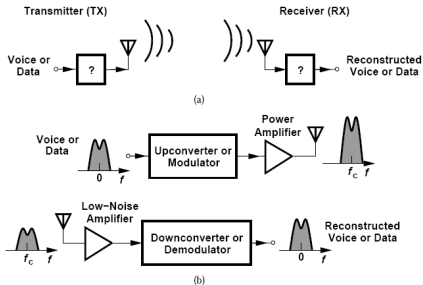
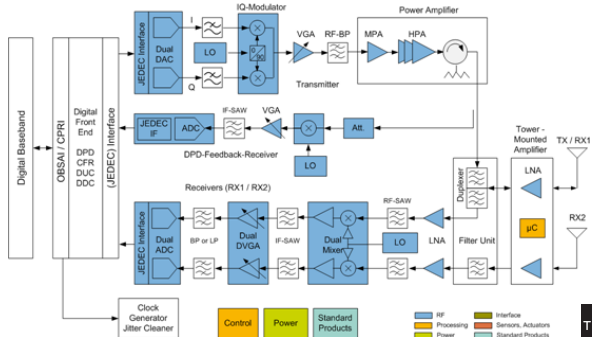
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- Former and current name (for 2026-2027)
  - FARE : Fundamentals of Analog and Radio Electronics
- Being renamed to :
  - DOMWiCS: Design of Modern Wireless Communication Systems



| Chapter | Title                                                   | Duration (TH) | Format | Instructors          |
|---------|---------------------------------------------------------|---------------|--------|----------------------|
| 1       | Elements of Communication Theory & Signals              | 4             | Lesson | G. PHAM              |
| 2       | Baseband Processing: Filtering & Data Conversion        | 2             | Lesson | C. JABBOUR           |
| 3       | Baseband Processing Simulation                          | 2             | Lab    | C. JABBOUR + G. PHAM |
| 4-a     | RF Front-End Blocks                                     | 1             | Lesson | K. NIOTAKI           |
| 5       | Frequency Synthesis & Signal Integrity                  | 2             | Lesson | P. DESGREYS          |
| 4-b     | Mixers                                                  | 1             | Lesson | C. JABBOUR + G. PHAM |
| 6       | System-Level Transceiver Specifications                 | 2             | Lesson | C. JABBOUR           |
| 7       | Software-Defined Radio (SDR) Hardware & Synchronization | 2             | Lesson | G. PHAM + C. JABBOUR |
| 8       | RF Front-End Simulation and Radio Implementation        | 3             | Lab    | G. PHAM + C. JABBOUR |

- Software-Defined Radio for Engineers, by Travis F. Collins, Robin Getz, Di Pu, and Alexander M. Wyglinski, 2018, ISBN-13: 978-1-63081-457-1.
  - Available online: [SDR4Engineers.pdf](#)
- Radio Frequency Integrated Circuits and Systems, 2nd Edition, by Hooman Darabi, 2020, Cambridge University Press.
- Radio Frequency Integrated Circuits and Technologies, by Frank Ellinger, 2008, Springer.
  - Available online (via Telecom VPN): [Radio Frequency Integrated Circuits and Technologies](#)
- Schaum's Outline of Theory and Problems of Analog and Digital Communications, by Hsu H. P., 2003, McGraw-Hill Education.

# Credits for Chapter 1 "Elements of Communication Theory & Signals"

## ■ Title slides have annotations

### ♣ reuses materials from

- PowerPoints for RF Microelectronics, Prepared by Bo Wen, UCLA
- based on : Behzad Razavi, *RF Microelectronics*.
- Available online : [Razavi, PowerPoints for RF Microelectronics | Pearson](#)

### ♠ reuses materials from

- RF transceivers short course, Rayan MINA, 2010

### ♦ reuses materials from

- Introduction to Digital Modulation, EE4367 Telecom. Switching & Transmission, Prof. Murat Torlak
- Available online : [lecturedm.pdf](#)

### ▲ reuses materials from

- Lecture 19 Basics of Wireless Communication, 6.976 High Speed Communication Circuits and Systems, Michael Perrott, MIT
- Available online : [lec19.pdf](#)

### ▼ reuses materials from

- BER calculation, Vahid Meghdadi
- based on *Wireless Communications* by Andrea Goldsmith
- Available online : [ber\\_awgn.pdf](#)

Permanent copies available in restricted access on [C2S website](#)

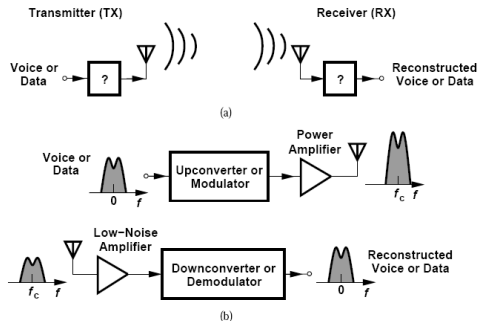
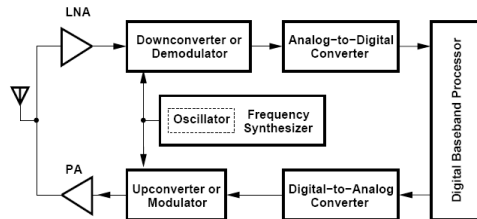
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| Slot | Title                                                                 |
|------|-----------------------------------------------------------------------|
| 1    | Signals, Spectrum, Sampling and Analog Modulation                     |
| 2    | Baseband Digital Transmission, Bandwidth Limitation and Pulse Shaping |
| 3    | Probability, Optimal Receiver and BER                                 |
| 4    | I/Q Passband Communication, OFDM and LTE basics                       |

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# Slot 1 – Signals, Spectrum, Sampling and Analog Modulation

From information to a modulated waveform



## Central idea

A communication system represents information by modifying (measurable) properties of a physical waveform.

By the end of this slot, you should be able to:

- describe communication signals in time and frequency
  - for AM, PM and FM;

## Worthnoting

No impairment is considered in this lesson slot : no noise, no distortion.

# What Is a Communication System?



At signal level:

$$r(t) = s(t) + n(t) \quad (1)$$

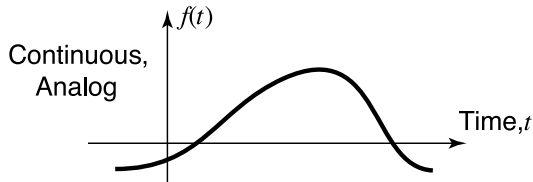
- $s(t)$ : transmitted waveform;
- $n(t)$ : unwanted disturbance or noise;
- $r(t)$ : received waveform.

**For now:** we concentrate on constructing and representing  $s(t)$ .

**Later:** Other slots ask how the receiver decides which information was transmitted from the noisy observation  $r(t)$ .

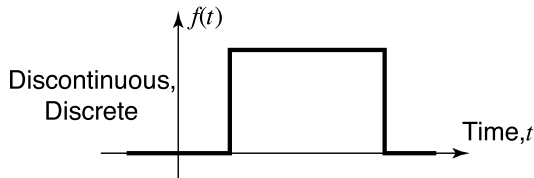
## ■ Analogue

- Ex: Voice, music, position,...



## ■ Digital

- Ex : piano notes, text (consisting of letters), RGB pixels



## ■ The information is given by the variation of a physical parameter = MODULATION

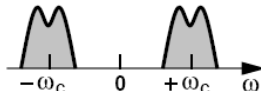
- Light, mechanical, electromagnetic waves

## Baseband modulation



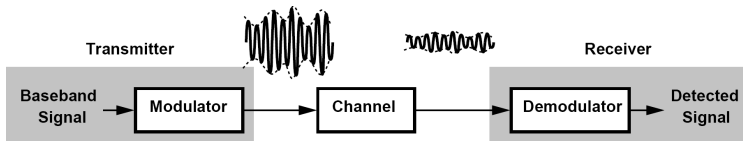
$$x(t) = a(t) \quad (2)$$

## Passband modulation



$$x(t) = a(t) \cos(\omega_c t + \theta(t)) \quad (3)$$

- For passband modulation : several parameters of a sinusoidal carrier can be modulated according to the baseband signal.



- A simple communication system consists of a modulator/transmitter, a channel, and a receiver/demodulator

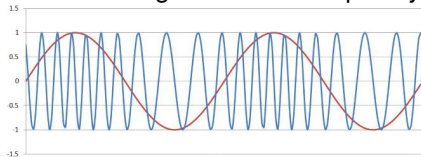
# Usual physical modulated parameters

## ■ Analogue modulation

- Amplitude

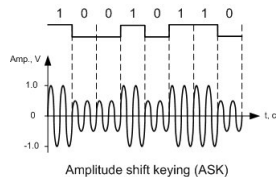


- Angle : Phase/Frequency

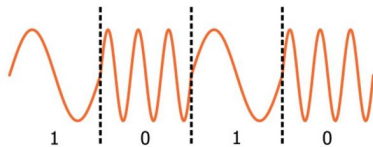


## ■ Digital modulation

- Amplitude



- Angle : Phase/Frequency

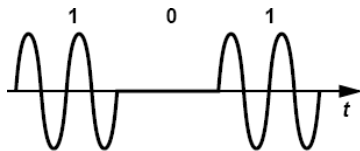


- Baseband
- Analog
- Amplitude
- Passband or Modulated carrier
- Digital
- Frequency / Phase

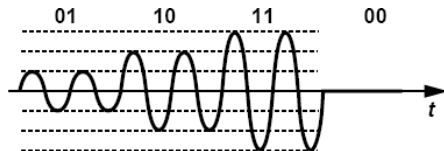
### The word modulation has several meanings

- In the context of electronics, modulation refers to the process of varying a carrier signal
  - Mental image : multiplication (mixing) of the baseband signal with the carrier signal (possibly using IQ form)
- In the context of communication theory (in particular in MATLAB), modulation refers to convert information to (complex-valued) symbols.

## Important aspects of modulations (♣)



2-level



4-level

### ■ Detectability

- the quality of the demodulated signal for a given amount of channel attenuation and receiver noise

### ■ Bandwidth Efficiency

- the bandwidth occupied by the modulated carrier for a given information rate in the baseband signal

### ■ Power Efficiency

- the type of power amplifier (PA) that can be used in the transmitter

# Continuous-Time and Discrete-Time Signals

A continuous-time signal is defined for every time  $t$ :

$$x(t), \quad t \in \mathbb{R}. \quad (4)$$

After sampling with sampling period  $T_s$ :

$$x[n] = x(nT_s), \quad n \in \mathbb{Z}. \quad (5)$$

The sampling frequency is

$$f_s = \frac{1}{T_s}. \quad (6)$$

## Important distinction

$x(t)$  and  $x[n]$  are different mathematical objects. Require appropriate mathematical (Fourier/Laplace or DT-Fourier/Z-transform) tools.

## Communication chain

Physical signals are continuous in time, while modern signal processing is predominantly discrete in time.

Consider

$$s(t) = A \cos(2\pi f_c t + \phi). \quad (7)$$

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$A$       Waveform amplitude.

$f_c$       Oscillation rate (Hz).

$\phi$       Phase at  $t = 0$  (radians).

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$T_c = \frac{1}{f_c}$       Period.

$P_s = \frac{A^2}{2}$       Normalized average power.

$\omega_c = 2\pi f_c$       Angular frequency (rad/s).

---

Amplitude, frequency and phase are directly observable properties of an oscillatory waveform.

# Three Natural Degrees of Freedom for Modulation

A carrier

$$s(t) = A(t) \cos(2\pi f(t)t + \phi(t)) \quad (8)$$

contains three obvious parameters that can carry information:

| Parameter     | Analog modulation | Later digital counterpart |
|---------------|-------------------|---------------------------|
| Amplitude $A$ | AM                | ASK / QAM                 |
| Frequency $f$ | FM                | FSK                       |
| Phase $\phi$  | PM                | PSK                       |

## Analog vs digital modulations

Analog modulation varies these parameters continuously.

Digital modulation will restrict the transmitter to a **finite set of allowed states** (i.e. values).

# Why Study Communication Signals in Frequency?

The time domain answers:

*How does the waveform evolve with time?*

The frequency domain answers:

*Where is the signal energy or power located in frequency?*

This matters because communication systems have:

- finite channel bandwidth;
- frequency-selective filters;
- allocated RF spectrum;
- interference from neighboring channels;
- sampling constraints determined by spectral support.

## Note

A waveform that appears simple in time may require a large bandwidth, while a complicated time-domain waveform can sometimes be spectrally compact.

For finite-power signals, the Fourier transform is:

$$X(f) = \int_{-\infty}^{+\infty} x(t) e^{-j2\pi ft} dt. \quad (9)$$

The inverse transform is:

$$x(t) = \int_{-\infty}^{+\infty} X(f) e^{j2\pi ft} df. \quad (10)$$

$$X(f) = |X(f)| e^{j\angle X(f)}. \quad (11)$$

with:

- magnitude  $|X(f)|$  (a.k.a. modulus);
- phase  $\angle X(f)$ .

The plot of  $10 \log_{10}(|X(f)|^2)$  is usually referred to as the **spectrum** of the signal.

For real  $x(t)$ :

$$X(-f) = X^*(f). \quad (12)$$

## Recall: Multiplication and Convolution

| Time domain convolution |                       | Time domain multiplication |                                            |
|-------------------------|-----------------------|----------------------------|--------------------------------------------|
| $x(t) * h(t)$           | $\longleftrightarrow$ | $X(f)H(f)$                 | $x(t)y(t) \longleftrightarrow X(f) * Y(f)$ |

An especially important communication result is<sup>1</sup>

$$x(t) \cos(2\pi f_c t) \longleftrightarrow \frac{1}{2} (X(f - f_c) + X(f + f_c)). \quad (13)$$

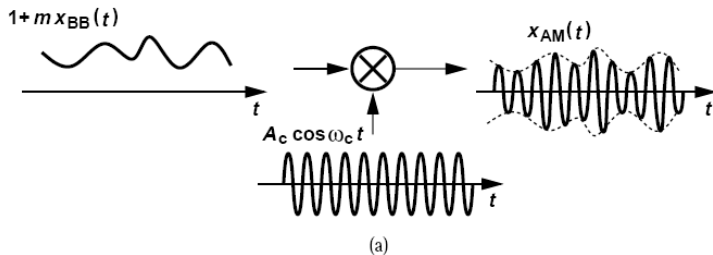
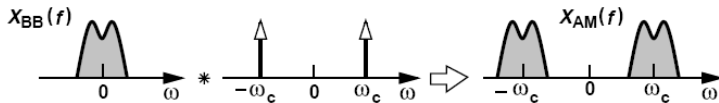
### Physical interpretation

Multiplying a baseband signal by a sinusoidal carrier **translates its spectrum** from around 0 Hz to around  $\pm f_c$ .

This simple property is the mathematical foundation of passband modulation.

<sup>1</sup>See slide 3 for a slightly more detailed derivation.

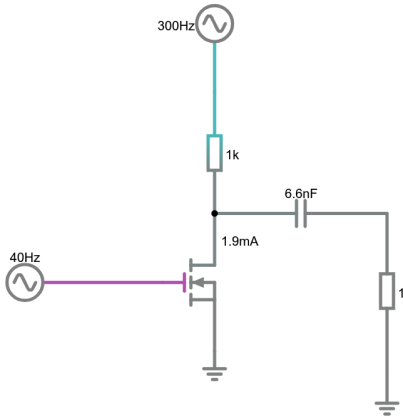
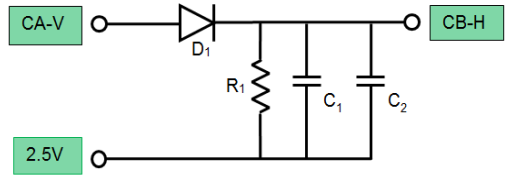
# Amplitude Modulation: graphical representation (♣)



$$x_{AM}(t) = A_c [1 + m x_{BB}(t)] \cos \omega_c t \quad (14)$$

■  $m$  is called the *modulation index*

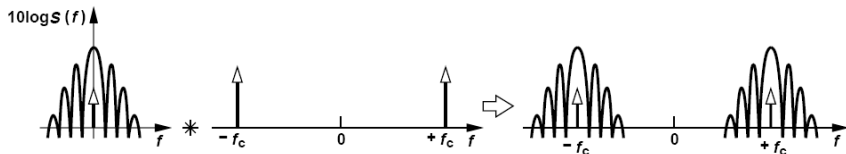
# Amplitude Modulation and demodulation: naive implementations



## Spectrum of (Binary) Amplitude Modulation (♣)

- Consider the product of a random binary sequence, toggling between 0 and 1, and a sinusoidal carrier
- The spectrum of a random binary sequence with equal probabilities of ONEs and ZEROS is given by

$$S(f) = T_b \left( \frac{\sin \pi f T_b}{\pi f T_b} \right)^2 + \frac{1}{2} \delta(f) \quad (15)$$



- Multiplication by a sinusoid in the time domain shifts this spectrum to a center frequency of  $\pm f_c$

You do not have the tools to derive this result rigorously at this point : the spectrum is obtained by the FT of the autocorrelation function of the random process.

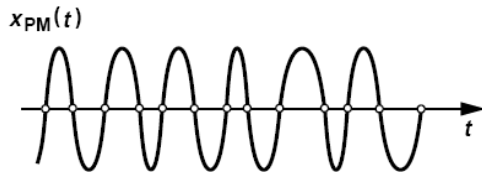
# Angle Modulation: Information in Carrier Angle

Write a constant-amplitude carrier as

$$s(t) = A_c \cos \theta(t), \quad (16)$$

with instantaneous phase

$$\theta(t) = 2\pi f_c t + \phi(t). \quad (17)$$



## Instantaneous Frequency

The instantaneous frequency is defined by

$$f_i(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt} \quad (18)$$

And with Eq. (17):

$$f_i(t) = f_c + \frac{1}{2\pi} \frac{d\phi(t)}{dt}. \quad (19)$$

PM and FM are closely related but not identical.

For phase modulation,

$$s_{\text{PM}}(t) = A_c \cos [2\pi f_c t + k_p m(t)] \quad (20)$$

where  $k_p$  has units of radians per unit of  $m(t)$ .

The instantaneous phase *deviation* is

$$\phi(t) = k_p m(t) \quad (21)$$

and the maximum phase deviation is

$$\Delta\phi = |\phi|_{\max} = \max_t |\phi(t)| \quad (22)$$

The instantaneous frequency for PM is then (start from Eq. (18)/(19)):

$$f_i(t) = f_c + \frac{k_p}{2\pi} \frac{dm(t)}{dt}. \quad (23)$$

Similarly, the maximum PM frequency deviation is

$$\Delta f_{\text{PM}} = \frac{k_p}{2\pi} \max_t \left| \frac{dm(t)}{dt} \right| \quad [\text{Hz}]. \quad (24)$$

## Physical interpretation

In PM, instantaneous frequency deviation depends on **how rapidly the message changes**, not only on its amplitude.

For frequency modulation,

$$s_{\text{FM}}(t) = A_c \cos \left[ 2\pi f_c t + 2\pi k_f \int_{-\infty}^t m(\tau) d\tau \right], \quad (25)$$

where  $k_f$  has units of Hz per unit of  $m(t)$ .

Differentiate the instantaneous phase:

$$f_i(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt} = f_c + k_f m(t). \quad (26)$$

Thus

$$\boxed{f_i(t) = f_c + k_f m(t)} \quad (27)$$

Obviously, the maximum FM frequency deviation is

$$\Delta f_{\text{FM}} = k_f \max_t |m(t)| \quad [\text{Hz}]. \quad (28)$$

## Physical interpretation

In FM, instantaneous frequency deviation depends directly on the amplitude of the message signal.

- Much more complicated than AM

$$p(t) = A_0 \cos[\omega_0 t + \Phi(t)] = \Re \left\{ A_0 e^{j\omega_0 t} e^{j\Phi(t)} \right\} \quad (29)$$

$$= \Re \left\{ A_0 e^{j\omega_0 t} \left( 1 + j\Phi(t) - \frac{\Phi(t)^2}{2!} + \dots + j^n \frac{\Phi(t)^n}{n!} + \dots \right) \right\} \quad (30)$$

- Closed form only possible for special cases
  - Sinus modulation
  - Small amplitude angle modulation
  - FSK

Let

$$m(t) = A_m \cos(2\pi f_m t). \quad (31)$$

The peak (frequency) deviation in  $f_i(t)$  is

$$\Delta f = k_f A_m. \quad (32)$$

The FM waveform becomes

$$s_{\text{FM}}(t) = A_c \cos \left[ 2\pi f_c t + \frac{k_f A_m}{f_m} \sin(2\pi f_m t) \right], \quad (33)$$

where

$$\beta = \frac{\Delta f}{f_m} = \frac{k_f A_m}{f_m} \quad [\text{dimensionless}] \quad (34)$$

is the FM modulation index.

$$\beta \ll 1 \quad (35)$$

narrowband FM behavior.

$$\beta > 1 \quad (36)$$

multiple significant sidebands.

## ■ Sinus modulation case

$$p(t) = A_0 \cos[\omega_0 t + \beta \sin(\Omega t)] \quad (37)$$

We use the associated complex exponential carrier:

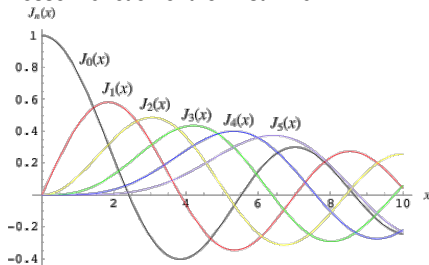
$$p(t) = \Re \left\{ A_0 e^{j[\omega_0 t + \beta \sin(\Omega t)]} \right\} \quad (38)$$

$$p(t) = \Re \left[ A_0 e^{j\omega_0 t} \sum_{n \in \mathbb{Z}} J_n(\beta) e^{jn\Omega t} \right] \quad (39)$$

with the use of the Fourier series expansion of  $e^{j\beta \sin(\Omega t)}$ ;  $J_n(\beta)$  is the Bessel functions of the first kind of order  $n$ . The modulated carrier  $p(t)$  can be written:

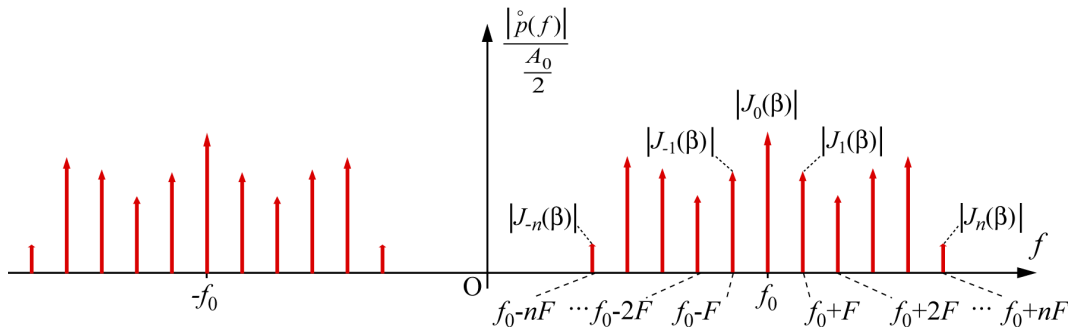
$$p(t) = A_0 \sum_{n \in \mathbb{Z}} J_n(\beta) \cos[(\omega_0 + n\Omega)t] \quad (40)$$

Bessel Function of the First Kind



# Spectrum of angle modulation : Sinus modulated case

$$p(t) = A_0 \sum_{n \in \mathbb{Z}} J_n(\beta) \cos[(\omega_0 + n\Omega)t] \quad (40)$$



Single-tone FM has spectral components at

$$f = f_c + nf_m, \quad n \in \mathbb{Z}. \quad (41)$$

Their amplitudes are proportional to Bessel coefficients  $J_n(\beta)$ .

The theoretical FM spectrum contains infinitely many sidebands.

For engineering purposes, Carson's rule estimates the occupied bandwidth:

$$B_{\text{FM}} \approx 2(\Delta f + B_m) = 2(k_f A_m + B_m) \quad (42)$$

It typically contains approximately 98% of the signal power for common FM conditions.

# AM, PM and FM: Synthesis

|                         | AM              | PM                                     | FM                          |
|-------------------------|-----------------|----------------------------------------|-----------------------------|
| Amplitude               | varies          | constant                               | constant                    |
| Phase                   | nominally fixed | controlled by $m(t)$                   | integral of $m(t)$          |
| Instantaneous frequency | $f_c$           | $f_c + \frac{k_p}{2\pi} \frac{dm}{dt}$ | $f_c + k_f m(t)$            |
| Bandwidth               | $2B_m$          | message dependent                      | $\approx 2(\Delta f + B_m)$ |

## Exercise 1 – Spectrum, Sampling and AM

Consider

$$m(t) = \cos(2\pi 10\,000t) + 0.5 \cos(2\pi 30\,000t). \quad (43)$$

It modulates a carrier using

$$s(t) = 2[1 + 0.6m(t)] \cos(2\pi 10^6 t) \quad [\text{V}]. \quad (44)$$

1. Determine the message bandwidth  $B_m$  and the positive-frequency spectral locations of the AM signal.
2. Determine the amplitude of each RF sinusoidal component and calculate the AM bandwidth.

## Exercise 1 – Solution I

The message contains frequencies 10 kHz and 30 kHz.  
Therefore

$$B_m = 30 \text{ kHz}. \quad (45)$$

Recall the trigonometric identity

$$\cos a \cos b = \frac{1}{2} [\cos(a + b) + \cos(a - b)]. \quad (46)$$

$$\begin{aligned} s(t) = & 2 \cos(2\pi f_c t) + 2 \times 0.6 \frac{1}{2} [\cos(2\pi(f_c - f_{10})t) + \cos(2\pi(f_c + f_{10})t)] \\ & + 2 \times 0.6 \times 0.5 \frac{1}{2} [\cos(2\pi(f_c - f_{30})t) + \cos(2\pi(f_c + f_{30})t)] \end{aligned}$$

The RF components are:

| Frequency              | Sinusoidal amplitude |
|------------------------|----------------------|
| 1.000 MHz              | 2 V                  |
| 1.000 MHz $\pm$ 10 kHz | 0.6 V                |
| 1.000 MHz $\pm$ 30 kHz | 0.3 V                |

Let

$$m(t) = 2 \cos(2\pi 5\,000t) \quad [\text{V}] \quad (47)$$

modulate a 100 MHz carrier.

$$\begin{cases} \text{PM: } \phi(t) = k_p m(t), \\ \text{FM: } f_i(t) = f_c + k_f m(t) \end{cases} \quad (48) \quad \begin{cases} k_p = 0.5 \text{ rad/V}, \\ k_f = 7.5 \text{ kHz/V} \end{cases} \quad (49)$$

1. For PM, determine the maximum phase deviation and the maximum instantaneous frequency deviation.
2. For FM, determine  $\Delta f$ , the modulation index  $\beta$ , and Carson's-rule bandwidth.
3. If the message frequency is doubled to 10 kHz while its amplitude remains 2 V, how do the PM and FM frequency deviations change?
4. Explain physically why PM and FM respond differently to this change.

## Exercise 2 – Solution I

For PM,

$$\phi(t) = k_p m(t). \quad (50)$$

The maximum phase deviation is

$$\Delta\phi = k_p A_m = \underbrace{0.5}_{[\text{rad/V}]} \times \underbrace{2}_{[\text{V}]} = \boxed{1 \text{ rad}}. \quad (51)$$

The instantaneous frequency is  $f_i(t) = f_c + \frac{k_p}{2\pi} \frac{dm(t)}{dt}$ . Since

$$\frac{dm(t)}{dt} = - \underbrace{2}_{A_m} (\underbrace{2\pi 5\,000}_{f_m}) \sin(2\pi 5\,000t), \quad (52)$$

the peak PM frequency deviation is

$$\Delta f_{\text{PM}} = \frac{k_p}{2\pi} \max_t \left| \frac{dm(t)}{dt} \right| = k_p A_m f_m = 0.5 \times 2 \times 5 \text{ kHz} = \boxed{5 \text{ kHz}}. \quad (53)$$

For FM,

$$\Delta f_{\text{FM}} = k_f A_m = \underbrace{7.5}_{[\text{kHz/V}]} \times \underbrace{2}_{[\text{V}]} = 15 \text{ kHz}. \quad (54)$$

Thus

$$\beta = \frac{\Delta f_{\text{FM}}}{f_m} = \frac{15}{5} = \boxed{3}. \quad (55)$$

Carson's rule gives

$$B_{\text{FM}} \approx 2(\Delta f_{\text{FM}} + B_m) = 2(15 + 5) = \boxed{40 \text{ kHz}}. \quad (56)$$

If  $f_m$  increases from 5 to 10 kHz:

$$\Delta f_{\text{PM}} = k_p A_m f_m : 5 \rightarrow 10 \text{ kHz}, \quad (57)$$

while

$$\Delta f_{\text{FM}} = k_f A_m = 15 \text{ kHz} \quad (58)$$

does not change.

**Reason:** PM responds to the derivative of  $m(t)$ ; FM responds directly to its amplitude.

## Slot 1 – What Should You Carry Forward?

- A signal must be understood jointly in the **time** and **frequency** domains.
- Carrier parameters

$$\boxed{A, \quad f, \quad \phi} \quad (59)$$

provide natural mechanisms for carrying information.

- AM modifies amplitude; PM modifies phase; FM modifies instantaneous frequency.
- Bandwidth is easy to compute for AM signals.
- Bandwidth is more complex to compute for PM and FM signals.

### Next problem – Slot 2

Instead of varying  $A$ ,  $f$  or  $\phi$  continuously, suppose the transmitter selects values from a **finite alphabet**.

# Appendix slides

Using the normalized  $1\Omega$  convention:

$$E_x = \int_{-\infty}^{+\infty} |x(t)|^2 dt \quad (60)$$

and

$$P_x = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x(t)|^2 dt. \quad (61)$$

**Finite Energy signal**

$$0 < E_x < \infty, \quad P_x = 0. \quad (62)$$

**Finite Power signal**

$$0 < P_x < \infty, \quad E_x = \infty. \quad (63)$$

Typical example: finite-duration pulse.

Typical example: sinusoid.

For a periodic signal:

$$P_x = \frac{1}{T_0} \int_{t_0}^{t_0+T_0} |x(t)|^2 dt. \quad (64)$$

$$p(t) = x(t) \cdot \cos(\omega_0 t) \quad (65)$$

$$P(f) = \left\{ X * \frac{1}{2} [\delta_{f_0} + \delta_{-f_0}] \right\} (f) \quad (66)$$

$$P(f) = \frac{1}{2} [X(f - f_0) + X(f + f_0)] \quad (67)$$

1.  $x(t)$  being real-valued, its TF has Hermitian symmetry

$$X(-f) = X^*(f) \quad (68)$$

The spectrum of the carrier  $p(t)$  is therefore symmetrical around the frequencies  $\pm f_0$ , with two bands lateral: an upper lateral band (ULB) and a lower lateral band (LLB).

2. A spectral ray at the carrier frequency is present if there is a DC component in  $x(t)$  (embedded carrier).

For a power signal, the power spectral density describes power distribution with frequency.

One deterministic definition is

$$S_x(f) = \lim_{T \rightarrow \infty} \frac{|X_T(f)|^2}{T}, \quad (69)$$

where  $X_T(f)$  is the Fourier transform of a finite-duration observation of the signal.  
For an LTI system

$$y(t) = x(t) * h(t), \quad Y(f) = H(f)X(f), \quad (70)$$

the PSD obeys

$$S_y(f) = |H(f)|^2 S_x(f) \quad (71)$$

**Engineering consequence:** a filter reshapes signal power according to its squared magnitude response.

# There Is More Than One Definition of Bandwidth

| Definition                 | Meaning                                                                                                                    |
|----------------------------|----------------------------------------------------------------------------------------------------------------------------|
| −3 dB bandwidth            | Frequency interval between points where power has fallen to one half of the reference value: $ H(f) ^2 =  H _{\max}^2/2$ . |
| Null-to-null bandwidth     | Frequency separation between the first spectral zeros surrounding the main lobe.                                           |
| Occupied bandwidth         | Smallest interval containing a specified fraction, for example 99%, of the total transmitted power.                        |
| Noise-equivalent bandwidth | Width of an ideal rectangular filter passing the same noise power as the actual filter.                                    |

For a low-pass filter using a one-sided convention,

$$B_{\text{NE}} = \frac{1}{|H(0)|^2} \int_0^{\infty} |H(f)|^2 df. \quad (72)$$

## Engineering rule

Never quote a bandwidth value without knowing which bandwidth definition is being used.