AMS and RF functions

M2 ICS - Scholar Year 2018-2019(S1)

ICS905 - Test

Duration 1h30 - Documents and calculator are allowed

Exercises

Exercise Power Amplifier Characterization 1

Exercise High Pass Filter 1

Exercise Digital modulations 1

All exercises are independent.

Direction de la Formation Initiale
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Exercise 1 - Power Amplifier Characterization

Question 1.1 For an amplifier, explain what the IP1 is.

Question 1.2 For an amplifier, explain what the IP3 is.

Question 1.3 Explain what the ACPR is.

Question 1.4 Explain what the PAPR is.

Question 1.5 What is the main consequence for an I-Q constellation when the signal is distorted ?

Exercise 2 - High Pass Filter

We would like to implement a high pass filter with a corner frequency of 10 MHz, with an attenuation higher than 30 dB at 1 MHz. The in-band attenuation should be lower than 1 dB in the band of interest (f>10 MHz).

Question 2.1 Plot the template of the wanted filter as well as the template of the equivalent low pass prototype.

Question 2.2 Calculate the order of the prototype filter for a polynomial approximation of Butterworth.

We decide to implement this filter using the following cell.

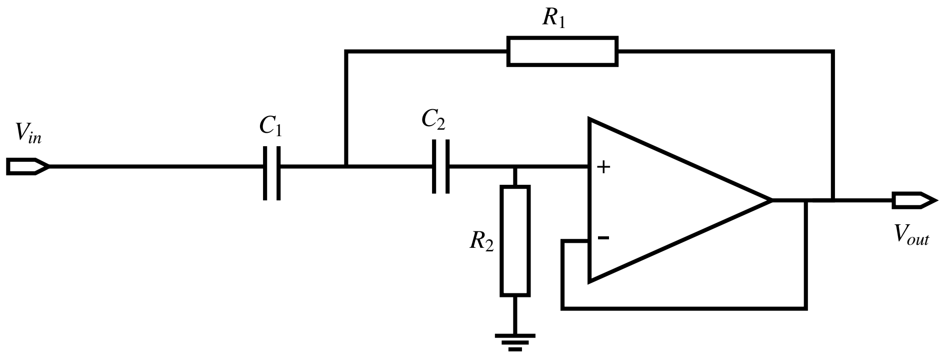


FIGURE 1 – High Pass Filter Implementation

Question 2.3 Calculate the transfer function in the laplace domain. The operationnal amplifier could be considered ideal.

We decide to set R_1=R_2=10 KΩ and $C_1 = C_2$ =10 pF.

Question 2.4 Does the filter meet to desired specifications ?

Exercise 3 - Digital modulations

3.1 Warming up – Generalities about OFDM

Question 3.1 What is the utility of cyclic prefix in OFDM systems and what is its requirement ?

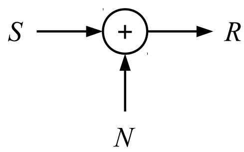
Question 3.2 What is the main disadvantage of OFDM from the transmitter circuit point of view ?

Question 3.3 According to you, is an OFDM-QPSK more immune to noise than a regular QPSK (assuming ideal transmit and receive chain) ?

Question 3.4 OFDM is for electrical or electromagnetic signals ; assume now the modulated signal is sound. Give examples of music instrument that could be used to transmit sound-OFDM and explain why. For each example, give the number of available subcarriers.

3.2 Warming up – Binary digital baseband communication system

Let us consider a simple binary communication system model. It is made up of a signal source which is modeled by a binary random variable S representing the message bit sent. The random variable S can take the values -1 and $+1$ with equal probability. This signal is degraded by an additive Gaussian noise which is represented by a Gaussian random variable N of zero mean and variance σ^2. The received value is a random variable $R = S + N$. This is illustrated in the figure below :



Question 3.5 Give the expression of $\mathbb{P}_S(s)$, the probability distribution of S, using Dirac delta distribution.

Question 3.6 Give the expression of $\mathbb{P}_{R|S}(r)$ the conditional probability density function of R given a value of $S = s$. [1]

Question 3.7 Find $\mathbb{P}_R(r)$ the probability density function of R. [2]

3.3 HIT [3] – Optimal threshold of a digital baseband communication system

To transmit a series of independent binary elements (a.k.a. bits) β, taking the values $\beta = 0$ or $\beta = 1$ with the same probability, we use an alphabet of two signals $h_0(t)$ and $h_1(t)$ of duration T, defined on the interval $[0, T[$. On the interval $[kT, (k+1)T[$, one transmits $h_0(t - kT)$ or $h_1(t - kT)$ according to the value of the binary element to be transmitted ; and for sake of simplicity, we will focus on the interval $[0, T[$. The two signals $h_0(t)$ and $h_1(t)$ are defined as follow :

$$h_0(t) = \begin{cases} A & \text{if } t \in [0, T/2[\\ 0 & \text{elsewhere} \end{cases} \quad ; \quad h_1(t) = \begin{cases} A & \text{if } t \in [T/2, T[\\ 0 & \text{elsewhere} \end{cases} \tag{1}$$

where A is a constant.

The signal is received in the presence of noise $b(t)$, white, Gaussian, additive, centered, independent of the signal and with bilateral spectral power density $N_0/2$.

We want to build a receiver formed by a filter with impulse response $g(t)$, followed by a sampler (at time t_0) and a threshold comparator, the filter being chosen to maximize the signal-to-noise ratio at the sampling instant t_0.

N.B. Several questions of this problem can be dealt independently.

Question 3.8 Draw a diagram of the communication system.

Question 3.9 Express the values u_0 and u_1 of the signal at the sampling instant, in cases where $h_0(t)$ and $h_1(t)$ have respectively been transmitted, assuming no noise. These two quantities will be expressed as integrals in which the fourier transforms $G(f)$, $H_0(f)$, $H_1(f)$ intervene.

Question 3.10 Give the expression of $y(t_0)$, the sampled value at the output of the receiver filter.

Question 3.11 Give the expression of $\mathbb{P}_{Y|U=u_0}(y)$ and $\mathbb{P}_{Y|U=u_1}(y)$ the conditional probability density functions of Y given a value of $U = u_0$ and $U = u_1$ respectively. The variance of the noise at the output of the receiver filter will be denoted σ^2.

The sample $y(t_0)$, taken at time t_0 at the output of the reception filter, is compared to a threshold S and a decision concerning the value of the binary element β is taken according to the following rule :

$$\begin{cases} y(t_0) > S & \text{then } \hat{\beta} = 1 \\ y(t_0) < S & \text{then } \hat{\beta} = 0 \end{cases} \tag{2}$$

where $\hat{\beta}$ represents the result of the decision.

1. The probability density of the zero mean gaussian random variable X is $\mathbb{P}_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \mathrm{e}^{-\frac{x^2}{2\sigma^2}}$
2. Remember the law of total probability : $\mathbb{P}(A) = \sum_n \mathbb{P}(A \mid B = b_n)\mathbb{P}(B = b_n)$
3. High-intensity training

Question 3.12 Explain which events are called "error".

Question 3.13 Given the conditional probability density functions of Y found in Question 3.11, sketch the conditional probabilities of the erroneous decision as areas for a threshold S closer to the value u_1 than u_0 :

$$P_{e0} = \int_S^{+\infty} \mathbb{P}_{Y|U=u_0}(y)\mathrm{d}y \tag{3}$$

$$P_{e1} = \int_{-\infty}^{S} \mathbb{P}_{Y|U=u_1}(y)\mathrm{d}y. \tag{4}$$

Question 3.14 Deduce from the previous sketch, the optimal threshold S_{opt} which minimizes the quantity $P_{e0} + P_{e1}$.

Finally, we can show that the probability of error with a *matched filter*[4] and the optimal threshold is :

$$P_{eb} = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{\Delta^2}{4N_0}} \tag{5}$$

with

$$\Delta^2 = \int_0^T (h_1(t) - h_0(t))^2 \, \mathrm{d}t \tag{6}$$

Question 3.15 What does Δ^2 represent from an algebraic geometry point of view ?

4. Filter whose response maximizes the signal to noise ratio