

# ICS905 - Fundamentals in AMS & RF Electronics (FARE)

M2 ICS - Scholar Year 2021-2022(S1)

# ICS905 - Exam

Duration 1h30 - Documents and calculator are allowed

## Exercises

Exercise	Low Pass Filter for Biomedical applications	1
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	All exercises are independent.	

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### Exercise 1 - Low Pass Filter for Biomedical applications

Biomedical devices are getting more and more popular as they allow a better monitoring of patient health and a higher efficiency for medical professionals. Biomedical signals are usually in the 10 mHz to 100 Hz frequency range. Processing these signals requires therefore low pass filters with very challenging low cut-off frequencies. In this exercise, we will study the design at system and circuit level of a filter for such applications.

The required filter will have a passband from DC to 100 Hz in which we allow a maximal attenuation of 1 dB. We also need an attenuation higher than 20 dB for all frequencies higher than 500 Hz<sup>1</sup>.

**Question 1.1** Determine the template of the needed low pass filter as well as the template of the corresponding prototype filter.

**Question 1.2** Calculate the order of the prototype filter for a polynomial approximation of Butterworth.



FIGURE 1 – Gm-C cell a)classical b) Miller enhanced

To implement the filter, we will use the cell of figure 1 a) as a basic cell. To meet the needed requirements, its cut-off frequency should be 158 Hz. Due to implementation and areas constraints,  $R_{out}^2$  and C could not exceed respectively 1 M $\Omega$  and 100 pF.

**Question 1.3** Calculate the transfer function of the cell. What is the lowest cut-off frequency that could be implemented with this filter?

To reduce the constraints, we will use the cell of figure b) instead. The amplifier A is a voltage amplifier with a high input impedance that we will consider infinite for the sake of simplicity. The relation between  $V_x$  and  $V_{out}$  is :

$$V_{out} = -A \cdot V_x$$

**Question 1.4** Determine the transfer function of the cell. Propose a set of values for C, A and  $R_{out}$  that allow to meet the needed cut-off frequency.

Question 1.5 Discuss briefly (3 lines) the advantages and drawbacks of this implementation

## Exercise 2 - RF front end

The subsections of this exercise (2.1, 2.2) can be treated independently.

<sup>1.</sup> In practice, the constraints are more challenging, they were simplified for the sake of simplicity.

<sup>2.</sup>  $R_{out}$  corresponds to the output impedance of the transconductance gm.

#### 2.1 System analysis

Question 2.1 Explain what is the 1 dB compression point for a power amplifier.

Question 2.2 Which is the role of a low noise amplifier at the receiver path of a transceiver?

Question 2.3 Which is the role of a duplexer in a transceiver?

Question 2.4 Calculate the overall gain (in dB) of the 2-staged power amplifier shown below :



The gain of the first power amplifier (PA 1) is G1 = 6.2 dB and the gain of the second power amplifier (PA 2) G2 = 4.5 dB.

**Question 2.5** Calculate the drain efficiency of a power amplifier operating at 1 GHz, where the output power is 0.5 W and the DC power is 10 W.

Question 2.6 Calculate the overall noise figure of the following topology :



The noise factor of the amplifier is  $5 \,\mathrm{dB}$ , the gain of the amplifier is  $10 \,\mathrm{dB}$  and the filter insertion loss is  $3 \,\mathrm{dB}$ .

#### 2.2 Units in RF

**Question 2.7** An amplifier senses a sinusoidal signal and delivers a power of  $-5 \,dBm$  to a load resistance of  $50 \,\Omega$ . Determine the peak-to-peak voltage swing across the load.

The amplifier is noisy and the system is represented in Fig. 2:



FIGURE 2 – Amplifier with a load resistance

Question 2.8 Which of the following diagram should be used to perform the noise calculations?



Question 2.9 From the chosen diagram, compute the average power of the noise transferred to  $R_L^{3}$ .

The power of the noise transferred to  $R_L$  is maximum when  $R_S = R_L^4$ .

**Question 2.10** Give the final expression of the maximum value of  $P_L$ .

**Question 2.11** We assume a bandwidth of 20 MHz and the system is at room temperature. Compute the SNR.

**Question 2.12** LTE specifies a receiver sensitivity of  $-91 \,\mathrm{dBm}$  for a bandwidth of 20 MHz. The detection of OFDM-QPSK with acceptable bit error rate  $(10^{-3})$  requires an SNR of about 2 dB. What is the maximum allowable RX noise figure?

### Exercise 3 - Elements of communication theory

The subsections of this exercise (3.1, 3.2, 3.3) can be treated independently.

### 3.1 Channel fading models



FIGURE 5 – RF channel models summary

Question 3.1 In the channel models summary in Fig. 5, what are the names of the variables :

- $-B_S;$
- $-B_C;$
- $-T_S;$
- $-T_C$ .

Clearly state the ones that refer to a channel feature or a signal feature.

<sup>3.</sup> You may want to ignore the signal voltage  $V_a$ 

 $<sup>4. \ {\</sup>rm the \ so-called} \ impedance \ matching$ 

#### 3.2Analytic signal

We consider the modulated signal :

$$x(t) = A(t)\cos\left(\omega_0 t + \varphi(t)\right) \tag{1}$$

**Question 3.2** Compute the Hilbert transform  $x_H(t)$  of x(t).

**Question 3.3** Compute the analytic signal  $x_A(t)$  of x(t).

#### 3.3Binary digital baseband communication system

Let us consider a simple binary communication system model. It is made up of a signal source which is modeled by a binary random variable S representing the message bit sent. The random variable S can take the values -1 and 1 with probability 0.7 and 0.3 respectively. This signal is degraded by an additive Gaussian noise which is represented by a Gaussian random variable N of zero mean and variance  $\sigma^2$ . The received value is a random variable R = S + N. This is illustrated in the following system diagram :



**Question 3.4** Based on the lesson or your intuition, sketch the density probability of R; the diagram must exhibit the particular features of the exercise (magnitudes, axes, ...).

Question 3.5 The probability law of S is a discrete law. However, for the rest of the exercise, we need to write it as a distribution. Give the expression of  $\mathbb{P}_{S}(s)$ , the probability distribution of S, using Dirac delta distributions  $\delta(\cdot)$ .

Question 3.6 Given that the probability density of N is :

$$\mathbb{P}_N(n) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{n^2}{2\sigma^2}},\tag{2}$$

give the expression of  $\mathbb{P}_{R|S}(r)$  the conditional probability density function of R given a value of S = s.<sup>5</sup>

**Question 3.7** Find  $\mathbb{P}_R(r)$  the probability density function of R.<sup>6</sup>

<sup>5.</sup> You can begin by writing  $\mathbb{P}_{R|S=0}(r)$  and  $\mathbb{P}_{R|S=1}(r)$ , and then  $\mathbb{P}_{R|S}(r)$ 6. Remember the law of total probability :  $\mathbb{P}(A) = \sum_{n} \mathbb{P}(A \mid B = b_n) \mathbb{P}(B = b_n)$